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PUZZLES – A CREATIVE WAY OF DEVELOPMENT OF LOGICAL THINKING

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Abstract

Logical thinking of students should be enhanced at all levels of their studies. There are many possibilities how to achieve it. In the paper one possible way within the subjects "Discrete Mathematics" and "Discrete Methods and Optimization" dealing with graph theory and combinatorial optimization will be presented. These mathematical disciplines are powerful tools for teachers allowing them to develop logical thinking of students, increase their imagination and make them familiar with solutions to various problems. Thanks the knowledge gained within the subjects students should be able to describe various practical situations with the aid of graphs, solve the given problem expressed by the graph, and translate the solution back into the initial situation.

Student engagement is crucial for successful education. Practical tasks and puzzles attract students to know more about the explained subject matter and to apply gained knowledge. There are an endless number of enjoyable tasks, puzzles and logic problems in books like "Mathematics is Fun", in riddles magazines and on the Internet. In the paper, as an inspiration, four puzzles developing logical thinking appropriate to be solved using graph theory and combinatorial optimization will be introduced. On these puzzles of different level of difficulty the students' ability to find out the appropriate graph-representation of the given task and solve it will be discussed as well.

The author of the paper has been prepared with her students various multimedia applications dealing with objects appropriate to subject matter for more than 15 years. In the paper we also discuss a benefit of multimedia applications used as a support of subjects "Discrete Mathematics" and "Discrete Methods and Optimization".

Key Words

Logical thinking, enjoyable teaching and learning, puzzles, Graph theory, Combinatorial optimization, multimedia application

Introduction

One of the pleasant ways to bring discussed topics closer to students and to practice them, is their illustrations and practical applications on real examples. Given terms or problems will be recollected well by students if they are presented in an interesting, sometimes „sprightly" example of life. To demonstrate the use of the discussed issues it is often worth including the appropriate puzzles (logical tasks) into teaching methods. Not just because logical tasks can provide students with an initial idea, and the motivation to apply the theoretical knowledge, but it can also greatly contribute to the development of students' logical thinking, their imagination, and, especially in the case of graph theory and combinatorial algorithms, enhance their ability to transfer the problem into a graph and solve it.

The least significant role in making learning more fun and easier to understand, is the use of multimedia applications, which can be used both by the teacher as a supplement to the problem interpretation and by students as an efficient assistance in their individual preparation.

In the paper we initially introduce the four principles that we apply in our teaching. Then we show, in four puzzles of different level of difficulty, how it is also possible to develop the logical thinking of students and to increase their imagination within the subjects dealing with graph theory and combinatorial optimization and how to engage students in the lessons. At the end we emphasize the role of suitable multimedia applications dealing with objects appropriate to course subject matter that support students' self preparation and teachers' explanations of a topic.

Effective education developing logical thinking and imagination

Logical thinking is an important foundation skill. Karl Albrecht (Albrecht, 1984) says *that the basis of all logical thinking is sequential thought. This process involves taking the important ideas, facts, and conclusions involved in a problem and arranging them in a chain-like progression that takes on a meaning in and of itself. To think logically is to think in steps*.

Let us add that *sequential thought can be enhanced through the development of algorithmic thinking and that algorithmic thinking can be deeply enhanced in the subjects dealing with combinatorial optimization*.

The main aim of the subjects "Discrete Mathematics" (DIMA) and "Discrete Methods and Optimization" (DMO) taught at the Faculty of Informatics and Management is to develop students' imagination and deepen their capacity for logical and algorithmic thinking. Let us introduce the subjects more detailed.

DIMA is a compulsory subject taught in the fourth term. Students gain a basic level of competence in graph theory and combinatorial optimization. In the DIMA lectures and lessons non-directed graphs have been discussed and to the theoretical background of each explained concept and problem enough time has been devoted. Concerning graph algorithms, we try to make students familiar with certain algorithms in contexts to be able to get deeper insight into each problem and entirely understand it. We always try to examine the given topic as thoroughly as possible and find a "bridge" to another topic. Well-prepared students should be able to describe various practical situations with the aid of graphs, solve the given problem expressed by the graph, and translate the solution back into the initial situation. Various logical puzzles serve as a very suitable tool for checking this ability (see thereinafter).

DMO is a compulsory subject taught in the seventh term. Its aim is not only to develop students' knowledge gained in the subject "Discrete Mathematics" and to focus on directed graphs, but also to enhance students' skills in self study of a given new part from the area of graph theory and combinatorial optimization and their ability to explain it to the others.

In the first half of the term lectures and lessons are organized in the similar way as lectures and lessons of the DIMA subject. However, the second half of the term is focused on enhancing students' skills in self-study. According to e.g. (Nowak, Gowin, 1984) or (Pascual, 2010) or (Huba, Pestún and Huba, 2011) who recommend creating environment encouraging students to take risk in classroom discussions, we let students some time for studying new matter and preparing presentations. They work in teams preparing also a presentation containing appropriate theory illustrated on examples. Defence and discussion with other students take place in remaining lectures and lessons.

An indivisible part of the DMO exam is a presentation on an optional topic from the area of the subjects DIMA or DMO describing a practical task. This part of the examination runs as a colloquium and each student shows his/her work to the colleagues taking part on the exam.

Our approach to the development of logical thinking of students within the above mentioned subjects can be characterized by the following basic principles that we apply in our teaching (Milková, 2009).

- When starting explanation of new subject matter, a particular problem with a real life example or puzzle is introduced and suitable graph-representation of a problem is discussed.
- If possible, each concept and problem is examined from more than one point of view and various approaches to the given problem solution are discussed.
- Visualization of the particular issue as well as it is possible is done.
- The explained topic it thoroughly practiced and students' examples describing the topic are discussed.

Material and Methods

Practical examples and *puzzles* serve as a motivation to the explained subject matter; they are good tool enabling students to get an idea about its use. But they are also very good tool for finding out if students are able *to describe a given task with the aid of graphs, i.e. find a graph-representation of the task, solve it and translate the solution back into the initial situation*. Particularly when solving puzzles it isn't always easy to find immediately the needed graph-representation.

We have been looking for problems in various sources (in real life, in books like "Mathematics is Fun", in riddles magazines, on the Internet) that can be efficiently solved with the help of graphs and introduce them into lectures devoted to the appropriate topic.

History

The history serves also as a good source of practical examples and puzzles. In the area of graph theory there is the very valuable book "Graph Theory 1736 – 1936" (Biggs, Lloyd and Wilson, 1976). The most important problems since 1736 till 1936 are introduced there (the problem called Seven bridges of

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Kőnigsberg formulated and solved by Leonhard Euler in 1736 is considered as the beginning of graph theory). In this book the connection of graph theory and puzzles is described well: *The origins of graph theory are humble, even frivolous. Whereas many branches of mathematics were motivated by fundamental problems of calculation, motion, and measurement, the problems which led to the development of graph theory were often little more than puzzles, designed to test ingenuity. But despite the apparent triviality of such puzzles, they captured the interest of mathematicians, with the result that graph theory has become a subject rich in theoretical results of surprising variety and depth.*

Logical tasks

The book written by Stanislav Vejmola (Vejmola, 1986) was the first impulse for our decision to include puzzles into the curriculum of the subjects DIMA and DMO. The book is organized in an interesting way. It is divided into three parts. The first part contains several problem assignments, in the second part the necessary graph theory background is introduced and, finally, in the third part solutions of the tasks given in the first part are shown.

In this paragraph let us introduce four puzzles of different difficulties, chosen from the Czech semi-monthly magazine *Hádanka a Křížovka* (*Riddle and Crossword puzzle* in English).

Logical tasks

Two detectives investigated the same group of people and used graph-representation of the relation between each pair of people who know each other. The first detective represented the people by letters, the other detective by numbers (see Fig. 1 and Fig. 2). Our task is to find out connection between their graph-representations.

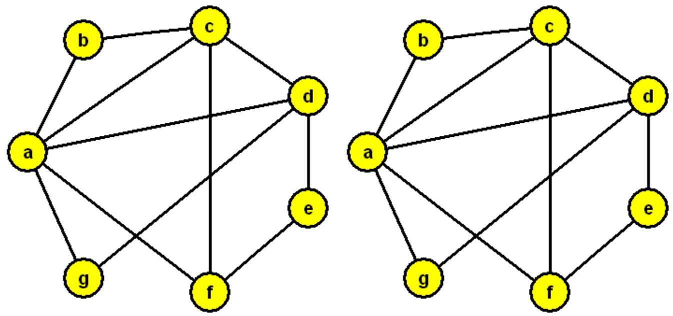


Fig. 1 The first detective graph-representation

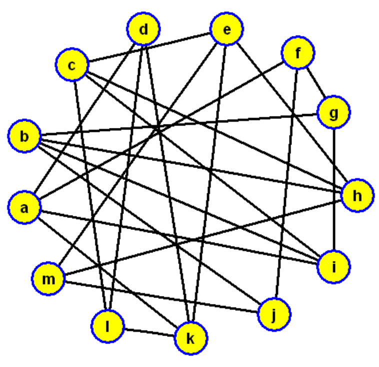
Fig. 2 The second detective graph-representation

This puzzle is obvious example suitable for motivation to the concept isomorphism and to solve it means to find the isomorphism between the two graphs given on Fig. 1 and Fig. 2.

Remark: Isomorphism is an important basic graph theory concept explained in any textbook dealing with graph theory. Its definition and use is described in an interesting way in (Matoušek and Nešetřil, 1998) for example.

Pins

There are 13 pins connected by strings in the way given on Fig. 3. Someone changed their positions as it is shown on Fig. 4. The task is to find the initial position of pins (the initial order of pins).



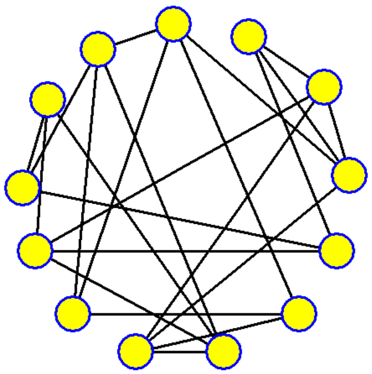


Fig. 3 Original position of pins given in the puzzle Pins

Fig. 4 Changed position of the pins

This puzzle is also an example suitable to the concept isomorphism. However, the two given graphs are more complicated and to find the isomorphism between them demands significantly more of students' attention. It is much easier and enjoyable to solve this puzzle using the GrAlg program (see thereinafter).

Towns

Try to place the names of towns Atlanta, Berlin, Caracas, Dallas, Lima, London, Metz, Nairobi, New York, Paris, Quito, Riga, Rome, Oslo, Tokyo into the frames of the given map (Fig. 5) so that no town shares any letter in its name with any towns in adjacent frames (neither horizontal nor vertical).

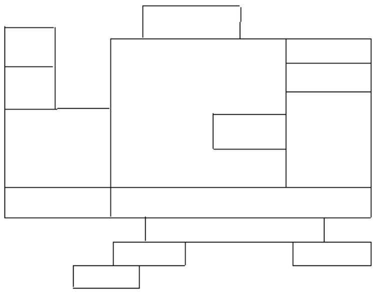


Fig. 5 Map of the puzzle Towns

To solve this puzzles using graph theory it is necessary to make graph-representation of both the map and also the relation between two towns that do not contain a same letter in their names at first (see Fig. 6 and Fig. 7), and then find the isomorphism between the graph representing the map and a subgraph of the graph representing the relation (Milková, 2009).

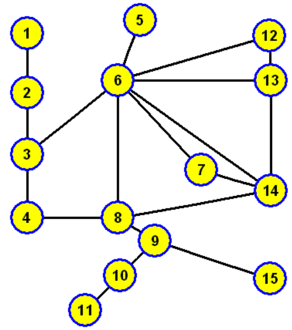


Fig. 6 graph-representation of the map given in the puzzle Towns

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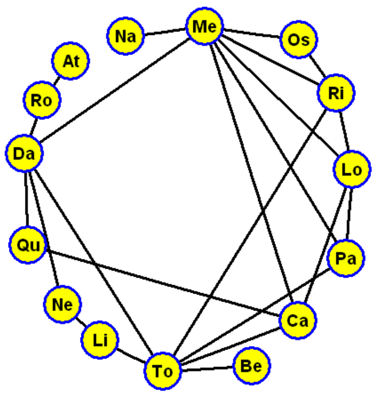
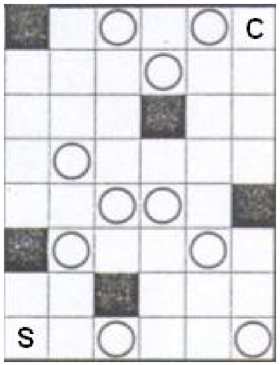


Fig. 7 graph-representation of the relation given in the puzzle Towns



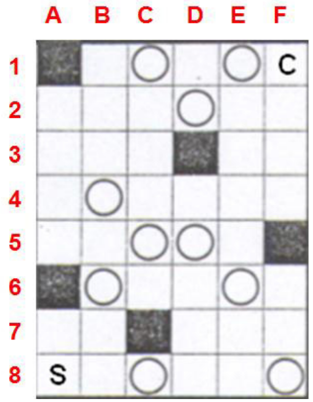


Fig. 8 Picture to the puzzle TWO and THRE

Fig. 9 Fig. 8 completed by numbers and letters

TWO and THREE

Let us consider the picture given in Fig. 8. There are three types of cells (fields); white, black and circular. The task is to find a way to move from point S to point C using the smallest number of steps possible while keeping the following rules:

- One step means to go on two (by the speed 2) or three (by the speed 3) cells.
- Go either horizontally or vertically.
- On S your speed is 2. As soon as you enter a circle, change the speed to 3 and as soon as you enter another circle, change the speed to 2 etc.
- Do not enter or go through black cells.

(Note: You can enter the same cells more times.)

Graph-representation of the picture on the Fig. 8 can be done in mind in the following way: Let us complete the picture on Fig. 8 by numbers and letters (see Fig. 9) and imagine that each cell is represented either by the vertex Pc^2, or by the vertex Pc^3, where P∈{A, B, C, D, E, F} and c∈{1, 2, …, 8}. The upper index determines the used speed.

In this way a directed graph G is obtained. Its vertices are Pc^i, P∈{A, B, C, D, E, F}, c∈{1, 2, …, 8}, i∈{2, 3}, and there is the directed edge from the vertex Xy^z to the vertex Uv^w in the graph G if and only if there exists a step from the vertex Xy^z to the vertex Uv^w defined by the above rules (i. e. there are for example edges ($A8^2$, $C8^3$), ($A8^3$, $D8^3$), ($C8^2$, $A8^2$), ($C8^2$, $E8^2$), ($C8^3$, $F8^2$), ($F8^2$, $D8^2$), ($F8^2$, $F6^2$)).

To solve the puzzle TWO and THREE means to use Breadth-First Search algorithm to find the shortest path from the vertex

A8^2 (the cell **S**) to the vertex F1^i (the cell **C**, which can be achieved either as the vertex F1^2 or as the vertex F1^3).

Remark: The Breadth-First Search algorithm as well as several other well-known algorithms finding the shortest path in an undirected or directed graph can be found in many books dealing with graph theory and combinatorial algorithms. An outline of them is given, for example, in the book Introduction to algorithms (Cormen, Leiserson, Rivest and Stein, 2009).

Multimedia applications created on a script given by the teacher with regard to students needs

Multimedia applications play an important role among the electronic study materials assigned to the appropriate subject. Along with large software products dealing with a wide spectrum of objects developed by a team of professionals there are various smaller presentations and programs dealing with objects appropriate to course subject matter created on a script given by the teacher with regard to students needs. The author of the paper has been interested in creation of such study material for many years. With her students she prepares large programs and presentations for more than one year, usually within their thesis. Students create smaller ones during the term.

We really agree with the text written in the paper (Williams, 2005), where Williams says *that students need images and visualization in addition to words. Science learning is about creating images in mind and teaching should support such image formation.*

With the help of multimedia applications students can revise the topic when it is needed; they can use them as a useful complement of the printed study text. Some programs and presentations offer complete revision of the large subject matter, another serve as detailed visualization of the given topic explained within the lecture. Short animations can serve teacher as an understandable motivation to the given topic.

In the DIMA and DMO subjects there is no problem in illustrating the needed concepts using graphs. However, it is very important to prepare suitable illustrative graphs and use colours to emphasize characteristics of the explained concepts and graph-algorithms.

Let us briefly introduce multimedia applications created for the discussed subjects and emphasize their main benefit for students' self-study and keeping the basic principles introduced in the section Effective education developing logical thinking and imagination.

Program GrAlg

The essential program developed for the DIMA and DMO subjects is the program GrAlg (Graph Algorithms) created in the Delphi environment by our student within his thesis (Šitina, 2010).

The main purpose of the application is the easy creation and modification of graphs and the possibility to emphasize with colours basic graph-concepts and graph algorithms on graphs created within the program.

The program enables the creation of a new graph, editing it, saving graph in the program, in its matrix representation and also saving graph in bmp format. It also makes it possible *to display some graph properties of the given graph represented by figure*, to add colour to vertices and edges, and to change positions of vertices and edges by "drop and draw a vertex (an edge respectively)".

The biggest advantage of the GrAlg program is the possibility to run programs *visualizing all of the subjects explained algorithms* on nondirected graphs in a way from which the whole process and used data structures can clearly be seen (see Fig. 10).

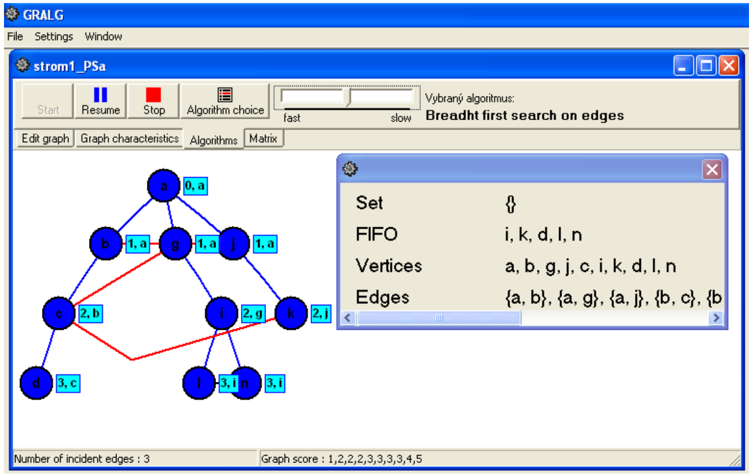


Fig. 10 Program GrAlg – visualization of the Breadth-First Search algorithm

The program allows the user to open more than one window so that two (or more) *objects or algorithms can be compared at once* (see Fig. 11).

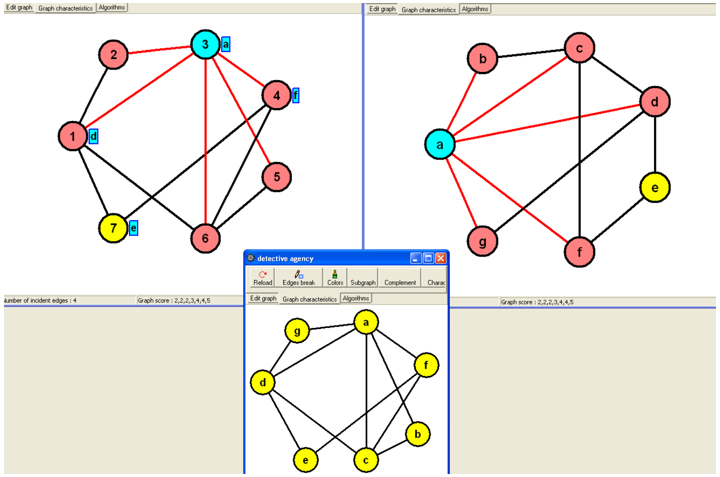


Fig. 11 Program GrAlg - three opened window to solve a puzzle Detective office

Using the GrAlg program students *can revise subject-matter* and more deeply understand it. They can use not only graphs prepared by the teacher but also graphs created by themselves and *explore the properties of these graphs and run in the program offered algorithms on these graphs*. The possibility to open more than one window enables them *to follow mutual relations among used concepts and algorithms*. The possibility to save each created graph in bmp format allows them easy insertion of needed graphs into their presentations (see the description of the DMO subject).

The GrAlg program is not only a substantial help to students in their self-study but it also helps teachers explain all needed concepts and the process of particular algorithms on lectures and seminars. More preciously, the program enables the teacher to complete his/her explanation within lectures in such a way that the topic is more comprehensible; the possibility to use colours allows the teacher to *emphasize needed objects and relations*; the option to open more than one window enables to *explain the problem from more points of view* and *show mutual relations among used concepts and algorithms*. Moreover, the possibility to save each created graph in bmp format allows teachers easy insertion of needed graphs into the study material and thus saves their time when preparing text material and presentations.

Results and Discussion

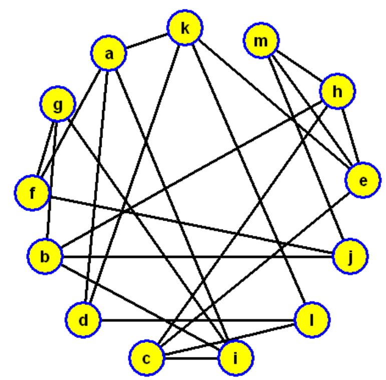
The theory of graphs and combinatorial optimization are wonderful, practical disciplines developing and deepening students' capacity for logical thinking. Using puzzles enable us to e*nhance logical thinking of students in an enjoyable creative way*.

Puzzles introduced as a motivation to the explained subject matter on lectures devoted to the appropriate topic aren't usually solved on the lecture. Students can (it isn't obligatory) solve them at home. However, there is often discussion on seminars about possible solutions.

The first two introduced puzzles don't demand graphical interpretation of the given task because it is set directly on graphs.

The map used in the third puzzle and also the demanded relation between two towns have obvious graph-representation for everyone experienced in graph theory. However to find out these graph-representations it makes students mostly difficulty because the puzzle *Towns* is placed to the DIMA subject at one of the first lessons. It serves as the very useful first step into the development of students' ability to „see" graph-representation of a task.

All tasks are easier to solve with help of the above mentioned program GrAlg. As we have mentioned the program enable to create graph and move its vertices and edges. Thus using the program there is no problem to change the view of the graph given on Fig. 3 to get graph on Fig. 4 (see Fig. 12) and to change the view of the graph given on Fig. 7 to get another picture (see Fig. 13), from which the solution is quite obvious.



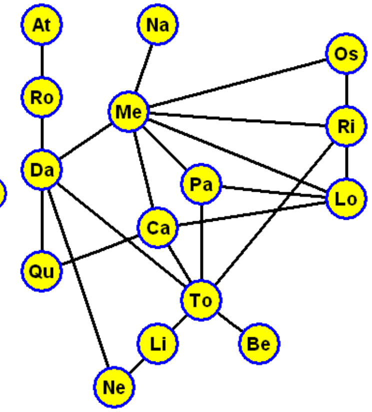


Fig. 12 another picture of the Fig. 3

Fig. 13 another picture of the Fig. 7

The fourth puzzle enhances students' imagination. Solving it students practice Breadth-First Search algorithm (see Fig. 10) and from an appropriate part of the Breadth-First Search Tree (Fig. 14) determine the shortest path.

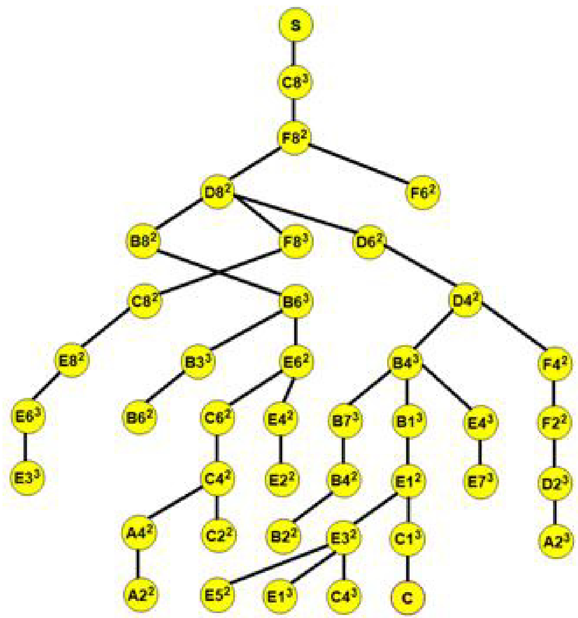


Fig. 14 Breadth-First Search Tree appropriate to the puzzle TWO and THREE

Moreover, the puzzle TWO and THREE evokes students to go on and find not only the one shortest path but also all the shortest paths between S and C. This topic is discussed with students within DMO subject. At students' disposal there is the paper (Milková, 2010a), where the description of the algorithm finding all the shortest paths between the two given vertices is described in detail. Description of the algorithm in English is done in the article (Milková, 2010b).

where the description of the algorithm finding all the shortest paths between the two given vertices is described in detail. Description of the algorithm in English is done in the article (Milková, 2010b).

We have already mentioned that we consider presentations and programs dealing with objects appropriate to course subject matter as a very useful complement of lectures and substantial help to students in their self-study. The main reason why we have devoted the time and energy to the creation of the program GrAlg although there are various programs visualising algorithms on graphs is that our program GrAlg is created exactly on a script given by the teacher (author of the paper) with regard to student's needs. For comparison let us mention e.g. very nice small open source Windisc, a collection of subprograms that deal with several discrete-math topics (see http://math.exeter.edu/rparris/windisc.html). The program *enables easy creation of graphs*, however *it does not visualise the whole process* but only the result of the selected algorithm. Another program, the program Algovision (author of the paper had an opportunity to revise a part of the program), was created by Luděk Kučera in Java environment (see kam.mff.cuni.cz/~ludek/Algovision/Algovision.html). The program serves as a support of his lectures given for students studying at the Mathematical-Physical Faculty, Charles University. The program contains several applets visualizing algorithms explained by professor Kučera on his lectures. Although the program is very useful, it is not user-friendly, applets are quite complicated, and therefore students need and have large manual with detailed description for users in their disposal. Moreover, *the program does not enable creation of own graphs*.

To have an own program created on a script given by the teacher with regard to students needs is really very beneficial. In near future we are going to prepare a text-book completed by CD containing the program GrAlg and appropriate graphs.

Conclusion

In the paper one possible way how to develop logical thinking of students and increase their imagination within the subjects dealing with graph theory and combinatorial optimization is presented.

On four puzzles of different level of difficulty were discussed the students' ability to find out the appropriate graph-representation of the given task and solve it.

Student engagement is crucial for successful education. Students learn more when they are intensively involved in their education, are asked to think about they are learning and apply it in different settings. Practical tasks and puzzles can help in this direction (see also e.g. (Hubálovský, 2010), (Hubálovský and Musílek, 2010), (Pražák, 2010), (Skiena, 1998)) as well as suitable multimedia applications.

Visualization of the particular issue as well as it is possible improves understanding of explained subject matter, enable the students to acquire, complete, test and deepen their knowledge and increase their imagination. Students admire quality multimedia applications prepared by their colleagues who, on the other hand, are proud that their works serve as a useful study material.

Acknowledgments

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