ERIES JOURNAL, VOLUME 11, ISSUE 3

CONTENT

Web pages: http://www.eriesjournal.com

Scientific journal of the Czech University of Life Sciences Prague JOURNAL ON EFFICIENCY AND RESPONSIBILITY IN EDUCATION AND SCIENCE, distributed by the Faculty of Economics and Management. Published quarterly. Executive editors: Ing. Martin Flégl, Ph.D. and Ing. Igor Krejčí, Ph.D., Editorial Office: ERIES Journal, Czech University of Life Sciences Prague, CZ 165 21 Prague 6 - Suchdol, Czech Republic, email: editor@eriesjournal.com, tel: +420 224 382 355.

ERIES JOURNAL

2018

ERIES Journal

JOURNAL ON EFFICIENCY AND RESPONSIBILITY IN EDUCATION AND SCIENCE

VOLUME 11

ISSUE 3

CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

An international peer-reviewed journal published by
Faculty of Economics and Management
Czech University of Life Sciences Prague

contact: editor@eriesjournal.com
www.eriesjournal.com
Online ISSN: 1803-1617
Printed ISSN: 2336-2375

EDITORIAL BOARD

Editorial Office

ERIES Journal, Czech University of Life Sciences Prague, CZ 165 21 Prague 6 - Suchdol, Czech Republic,
email: editor@eriesjournal.com,
tel: +420 22438 2355

Registration number: MK ČR E 21414

INSTRUCTIONS FOR AUTHORS

The Journal on Efficiency and Responsibility in Education and Science publishes papers of the following categories: full research papers, short communications, review studies and book reviews (on invitation only).

- FULL RESEARCH PAPERS
- SHORT COMMUNICATION
- REVIEW STUDY

Papers are published in English. A paper may comprise an empirical study using an acceptable research strategy, such as survey, case study, experiment, archival analysis, etc. It may contain a theoretical study aimed at advancing current theory or adapting theory to local conditions or it may arise from theoretical studies aimed at reviewing and/or synthesizing existing theory. Concepts and underlying principles should be emphasized, with enough background information to orient any reader who is not a specialist in the particular subject area.

Submission checklist

The paper. The paper is carefully formatted according to the template of the journal (see bellow). Special attention is paid to the exact application of the Harvard referencing convention to both continuous citations and list of references. If an electronic source has the DOI number assigned, also it will be provided in the list of references. Manuscripts are submitted via the editorial system in the DOC.

Research highlights. The core results, findings or conclusions of the paper are emphasized in 1-3 bullet points (max. 100 characters per bullet point including spaces). The highlights are submitted as a text into the submission form in the editorial system.

Copyright form. The submission of a paper will imply that, if accepted for publication, it will not be published elsewhere in the same form, in any language, without the consent of the Publisher. The manuscript submitted is accompanied by the copyright form signed by the corresponding author who declares the agreement of all authors with the conditions in the Form. The Form is submitted into the editorial system in the PDF format.

Suggested reviewers. It is required to suggest two experts appropriate to evaluation of the paper. The experts should be out of the affiliation of the author(s), Czech University of Life Sciences Prague, and also both experts should be from different affiliations. The reviewers are submitted into the text fields in the submission form of the editorial system.

Preparation of the manuscript (technical notes)

Authors are responsible for applying all requirements that are specified in the journal's paper template in individual sections. Especially, the paper must provide a short review of current state in the area of the paper's aim in Introduction. The paper should refer significant sources, particularly scientific journals or monographs.
Papers must be closely scrutinized for typographical and grammatical errors. If English is not author's first language then the paper should be proof-read by a native English-speaking person, preferably one with experience of writing for academic use. Spelling should follow the Oxford English Dictionary.
Tables, graphs and illustrations should be drawn using a suitable drawing package. Colour may be used. Place all diagrams and tables where you wish them to appear in the paper. Ensure your diagrams fit within the margins and are resizable without distortion.

Review procedure

Following Editorial recommendation, papers are submitted to a double-blind peer review process before publication. Commentary by reviewers will be summarized and sent by email to authors, who can choose to revise their papers in line with these remarks. Re-submitted papers should be accompanied by the description of the changes and other responses to reviewers' comments (see above), so that the desk-editor can easily see where changes have been made.

Copyright

Authors are fully responsible for the paper's originality and for correctness of its subject-matter, language and formal attributes. Author's statement should be enclosed declaring that the paper has not been published anywhere else.

The submission of a paper will imply that, if accepted for publication, it will not be published elsewhere in the same form, in any language, without the consent of the Publisher. Before publication, authors will be asked to complete a copyright release, giving the publisher permission to publish the paper in a specific issue of this Journal. Overall copyright ownership of the paper, however, remains with the author/s. It is the authors' responsibility to obtain written permission to quote material that has appeared in another publication.

EDITORIAL

We are pleased to introduce the third issue of the year 2018 (vol. 11, no. 3). The connecting topic of the published articles can be related to quality in education. All education institutions from the elementary until the tertiary education should provide an education that attracts students' interests and responds to current labour market needs. To achieve this, the education institutions should continuously try to adapt and improve their study programs, as well as teaching methods, to current educative trends. This can be achieved by introducing new teaching methods and tools into classrooms. Similarly, it is of a high importance to understand reasons for lower students' academic results, which can lead to unsuccessful termination of studies. Similarly to business organisations, many education institutions are in a competitive environment. Therefore, the ability of innovation in education is a crucial factor for future success.

The first article "Assessing future teachers' knowledge on fractions: Written tests vs concept cartoons", by author Libuše Samková deals with the possibilities of Concept Cartoons usage in future teachers' education. The author conducted an empirical study with 23 future primary school teachers from the Czech Republic and 44 future primary school teachers from Slovakia divided into two groups. The author used a standard written test which included four different word problems (T1-4) with an increasing level of difficulty. Participants in the first group had to solve all four problems with a requirement to solve them within the framework of primary school mathematics. The participants in the second group had to solve only the T3 problem with no restrictions nor recommendations on the solution procedure. In both cases, the participants obtained a bubble-dialogue picture related to the T3 problem. In both groups, the comparison of results and solution procedures revealed that many participants who mastered the word problem(s) displayed a fundamental misconception when working with the Concept Cartoon. Concept Cartoons can be valuable in future teacher education. However, the important question is how to employ them in the education.

The second article "Mathematical problem-solving strategies among student teachers" from Melanie Guzman Gurat seeks to enhance our understanding of the mathematical problem-solving strategies among student teachers. The participants of the study were 23 student teachers enrolled in Problem-solving course at Saint Mary's University during the 2011 summer term. In the first part of the study, the participants answered a set of problem-solving tasks and the Mathematics Motivated Strategies Learning Questionnaires. The results of the first part were analysed in order to construct a guidance for the second part of the study. In the second part, semi-structured interviews were conducted, which were later transcribed to explore problem-solving strategies. The obtained results revealed that the student teachers mainly use cognitive and metacognitive problem-solving strategies. The findings also suggest a significant influence of the strategies on the academic performance of the student teachers.

The third article "Students who have unsuccessfully studied in the past – Analysis of causes" by Petr Mazouch, Veronika Ptáčková, Jakub Fischer and Vladimír Hulík deals with an analysis of students in tertiary education who did not finish their tertiary education, however, decided to re-enrolled to studies after some time. The authors try to discover what social and demographic factors (such as the type of high school, gender, parents' social status, highest achieved education, among others) influence students' decision to change studied university or field of study. The analysis is based on the responses from 16,653 students in the EUROSTUDENT VI survey. The authors used decision trees and binary logistic regression methods to observe the significance of the analysed factors. The authors observed that the satisfaction with the university is a key classifier for drop-out. The type of secondary school studied was the second major factor. In this case, students who come from grammar schools or continue to study in the field of study from a specialized high school have a better chance of completing tertiary studies successfully. The third most significant factor is the student's social background.

We would like to thank all reviewers who contributed to this third issue of 2018, as well as we would also like to thank all authors who have submitted their manuscripts to ERIES Journal. We hope that all our readers will find this issue interesting, and we also hope that ERIES Journal will continue contributing to the field of efficiency and responsibility in education with new insights, research methods and analyses as it has contributed so far.

Martin Flégl
Executive Editor
ERIES Journal

https://www.eriesjournal.com
https://www.linkedin.com/company/eriesjournal/
https://erie.pef.czu.cz

ASSESSING FUTURE TEACHERS' KNOWLEDGE ON FRACTIONS: WRITTEN TESTS VS CONCEPT CARTOONS

Libuše Samková

Faculty of Education, University of South Bohemia, Jeronýmova 10, České Budějovice, Czech Republic, lsamkova@pf.jcu.cz

Highlights

- *Concept Cartoons – a tool for observing future primary school teachers' knowledge*
- *They can provide us with information that might not be obtained through standard written tests*

Abstract

The contribution investigates opportunities that an educational tool called Concept Cartoons can offer in future teachers' education, namely in comparison with word problems in standard written tests. The referred empirical study was conducted in two separated consecutive stages, with two groups of future primary school teachers (the first one from the Czech Republic, and the second one from Slovakia). The participants of the first stage solved four word problems (T1, T2, T3, T4) with increasing difficulty within the written test, and a problem with a similar structure and difficulty as T3 but in the Concept Cartoon form. The second stage of the study served as a complementary stage, its participants solved only the word problem T3 and the Concept Cartoon. In both stages, the comparison of results and solution procedures revealed many participants who mastered the word problem(s) but displayed a fundamental misconception when working with the Concept Cartoon. Two thirds of the participants presented non-corresponding responses to these two corresponding tasks: they solved one of them correctly and the other one incorrectly. All of the problems in the study were based on the part-whole interpretation of fractions, the revealed misconception consisted of incorrect determination of the whole.

Article type

Full research paper

Article history

Received: May 29, 2018

Received in revised form: September 5, 2018

Accepted: September 8, 2018

Available on-line: September 30, 2018

Keywords

Concept Cartoons, fractions, future primary school teachers, problem solving, word problems

Samková L. (2018) "Assessing Future Teachers' Knowledge on Fractions: Written Tests vs Concept Cartoons", *Journal on Efficiency and Responsibility in Education and Science, Vol. 11, No. 3*, pp. 45-52, online ISSN 1803-1617, printed ISSN 2336-2375, doi: 10.7160/eriesj.2018.110301.

Introduction

During university preparation, mathematics content knowledge of future teachers is often assessed through standard written tests where future teachers solve various calculation tasks and word problems. In my recent work, I have studied an educational tool called Concept Cartoons and its opportunities in assessing various aspects of future teachers' knowledge, e.g. their way of grasping situations related to word problems (Samková and Tichá, 2015), the openness of their approach to mathematics (Samková and Tichá, 2016b), their reasoning (Samková and Tichá, 2017b), problem posing skills (Samková and Tichá, 2016b, 2017b), informal foundations of pedagogical content knowledge (Samková, 2018a). One of the first studies (Samková and Tichá, 2015) showed how problems assigned in the Concept Cartoon form might become a valuable alternative to standardly assigned word problems. It also revealed one future teacher who incorrectly solved an unequal partition problem assigned in the Concept Cartoon form but correctly solved its alternative version assigned as a word problem. A subsequent interview of this future teacher showed that she had just learned the method needed for solving unequal partition word problems by rote, without understanding. And so, naturally, a question arose whether this happened as an exceptional case or whether Concept Cartoons could generally provide us with information on mathematics content knowledge that might not be obtained by word problems in standard written tests. This question led to a qualitative empirical study that will be reported here.

As in previous studies, the here reported study will focus on future primary school teachers, i.e. future teachers for pupils from 6 to 11 years of age. The mathematical topic in the centre of the study will be the topic of fractions, namely the part-whole interpretation of fractions. The topic of fractions is very important for future primary school teachers, there are many empirical studies reporting that the topic belongs to the most difficult ones for pupils (Lamon, 1999; Ryan and Williams, 2011; Steffe and Olive, 2010) as well as for future teachers and teachers (Cramer and Lesh, 1988; Ma, 1999; Depaepe et al., 2015; Singer, Ellerton and Cai, 2015). Drawing on this fact, the topic of fractions often plays a significant role in studies investigating questions related to partial understandings, sources of learner misconceptions and error-handling practices of teachers. For instance, Kazemi and Stipek (2001) employ the topic of fractions to illustrate their results on how to use errors to reconceptualise problems, explore contradictions and pursue alternative strategies of teaching, Schleppenbach et al. (2007) employ it to discuss the opportunities to create an error-friendly environment in the classroom. This brings us to the possibility to enable errors as opportunities for learning (Ingram, Pitt and Baldry, 2015), to the question of what is the role of misconceptions in the classroom (Nesher, 1987) and back to the idea of Concept Cartoons as one of the ways how partially represent the classroom environment to future teachers and teachers by presenting them possible pupils conceptions and misconceptions. Such an arrangement links this contribution also with the topic of teachers' ability to notice (van Es and Sherin, 2002; Star and Strickland, 2008), especially with the issue of noticing mathematics specific phenomena (Vondrová and Žalská, 2015).

From the perspective of ERIE conferences and ERIES Journal, the topic of the contribution is related to educational issues like students' solving strategies (Novotná and Vondrová, 2017) and knowledge-based reasoning (Uličná, 2017). It directly follows the issues presented by me and my colleague at the two previous

ERIE conferences (Samková and Tichá, 2016a, 2017a), and in the last two volumes of the ERIES Journal (Samková and Tichá, 2016b, 2017b).

This paper has been developed as an extension of the contribution (Samková, 2018b). I took advantage of my recent internship in a neighbouring country and enriched the research published in (Samková, 2018b) with further data.

The text is organized as follows: at the beginning, it presents word problems on fractions in the framework of the primary school curriculum and introduces participants of the study and the employed tools (written tests, Concept Cartoons). Then it describes the course of data collection and data analysis, presents findings, and discusses them.

Fractions at the primary school level

The study reported here concerns two neighbouring countries with a partly shared history of education: the Czech Republic and Slovakia. At the primary school level in the Czech Republic, the topic of fractions consists of the concept of a fraction per se, which is fundamentally interpreted through part-whole or part-part interpretations. The part-whole interpretation is based on partitioning either a continuous quantity or a set of discrete objects into equal-sized subparts or subsets (NÚV, 2015; Behr et al., 1983; Lamon, 1999). In word problems, pupils usually deal with requirements to ascertain a fractional part for a given whole, a whole for a given fractional part, or a complement of a given fractional part to a whole. In more difficult tasks they also deal with requirements to ascertain a fractional part when another fractional part with the same whole is given. As the time goes, the pupils meet more complex tasks combining several different fractional parts (either with the same whole, or with different wholes), and also tasks that are based on one or more fractional changes (each of them is applied either on a whole or on a part). To solve such tasks successfully, the pupils as well as the teacher have to be well oriented in the situation described in the word problem and have to decide properly about the parts and the whole in the situation. For task samples and their attributes see Table 1.

The tasks in Table 1 are sorted by difficulty, having T3 identified as more difficult than T2 since tasks with fractional changes generally appear to trigger much more errors and misconceptions than tasks without them. The reasons probably come from two sources. First, the errors and misconceptions might relate to linguistics: the tasks without a fractional change often use the preposition “of” in the text to refer to the whole in the described situation (4/7 of *all pupils*, 3/8 of *his potatoes*, 4/5 of *the rest*), but the tasks with the fractional change do not use this preposition, and so the decision about the whole is much more complicated there. In Czech and Slovak languages (which are the languages of the study participants), the syntax of these matters differs only a little from the English syntax, and the core of the problem stays the same. The second source of errors and misconceptions might consist in the fact that the concept of a fractional change combines additive and multiplicative structures together, and this combination results in the absence of symmetry that would be present if the structures were treated separately: the statement “A is 4 more than B” that expresses an additive structure describes the same situation as the statement “B is 4 less than A”, the statement “A is 4-times more than B” that expresses a multiplicative structure describes the same situation as the statement “B is 4-times less than A”, but the statements “A is 1/4 more than B” and “B is 1/4 less than A” that express a structure related to a fractional change describe two diverse situations – the first situation having B as the whole and the latter one having A as the whole, so that the quarters are not equal. Such asymmetry is also reported by Lamon (1999) in so called shrinking and enlarging within the topic of percentages: here the statement “A is 25 % more than B” does not correspond to the statement “B is 25 % less than A”. For more about linguistic issues related to fractional changes and different structure and difficulty levels of tasks on fractions at primary school level in the Czech Republic see (Samková and Tichá, 2017b).

T1	There are 16 girls in our class, which is 4/7 of all pupils. How many boys are there?	one whole, two different fractional parts, one fractional part being a complement of the other, requirement to ascertain a fractional part when another fractional part with the same whole is given, the number in the assignment is not the whole, the text uses the preposition “of” to refer to the whole
T2	A greengrocer came to a market for two days. On Monday he sold 3/8 of his potatoes, on Tuesday 4/5 of the rest. How much of the potatoes was not sold? How many kilograms of potatoes did the greengrocer bring to the market provided he sold 200 kilograms on Tuesday?	two different wholes, one of the wholes is a complement of a fractional part to the other whole, requirement to ascertain a fractional part for a given whole, requirement to ascertain a whole for a given fractional part, the number in the assignment is none of the wholes, the text uses the prepositions “of” to refer to the wholes
T3	A bookseller discounted the price of a book by a quarter to 60 crowns. How many crowns did the book cost before the discount?	one whole, one fractional change (decrease) of the whole, requirement to ascertain the state before the change for a given state after the change and a given fractional change, the number in the assignment is not the whole, the text does not use the preposition “of” to refer to the whole
T3*	Today’s audience at the athletic stadium equals 8000. It’s a quarter more than yesterday. What was yesterday’s audience?	one whole, one fractional change (increase) of the whole, requirement to ascertain the state before the change for a given state after the change and a given fractional change, the number in the assignment is not the whole, the text does not use the preposition “of” to refer to the whole
T4	A breeder keeps rabbits. Currently, 1/3 of his rabbits are white, and the others are grey. The breeder plans to give 3 grey rabbits to his neighbour today, and get 3 white ones for exchange. After this exchange, the proportion of white rabbits will rise to 4/9. How many rabbits does the breeder have?	one whole, two changes (decrease, increase) of two fractional parts that complement each other, requirement to ascertain the whole from a given state of one of the fractional parts before the change, a given state of this fractional part after the change and a given change, the number in the assignment is not the whole, the text uses the preposition “of” to refer to the whole

Table 1: Samples of various word problems on fractions, increasing code numbers in the first column refer to increasing difficulty of the problems; attributes of the problems are listed in the last column

In Slovakia, the topic of fractions is present in the primary school curriculum only at the propaedeutic level, mainly in the part-whole interpretation: in the sense of halving, thirding or quartering a given whole, and of ascertaining a whole for a given half, third or quarter (Švecová et al., 2017).

Nevertheless, in both countries future primary school teachers meet the topic of fractions in its entirety (i.e. all interpretations of fractions including ratios and percentages, and operations on fractions) during their teacher preparation content courses. And, like elsewhere in the world, they tend to provide misconceptions on the topics, especially on issues related to the part-whole interpretation (Hošpesová and Tichá, 2015; Samková and Tichá, 2017b; Pavlovičová and Švecová, 2017).

Materials and Methods

This study addresses the research question "Can Concept Cartoons provide us with information on mathematics content knowledge that might not be obtained through word problems in standard written tests?"

Participants

The research was conducted with two groups of participants, university students of the master degree program for future primary school teachers. The first group consisted of 23 future primary school teachers from the University of South Bohemia in České Budějovice, Czech Republic, and the second group consisted of 44 future primary school teachers from the Constantine the Philosopher University in Nitra, Slovakia. In both cases, I worked with completely all students that came to the compulsory lesson where data were collected.

Diagnostic instruments

As diagnostic instruments in my study, I used a standard written test and a Concept Cartoon. The written test included four word problems with increasing difficulty: T1, T2, T3 and T4 from Table 1. The participants from the first group had to solve all four tasks, with a requirement to solve them within the framework of primary school mathematics[1] (i.e. they were not allowed to use unknowns and equations in their solution procedures, nor topics outside primary school mathematics such as percentages). The participants from the second group had to solve only the task T3, no restrictions nor recommendations on the solution procedure were communicated to them.
With the Concept Cartoon, all the participants obtained a bubble-dialogue picture related to the task T3* from Table 1; the picture is presented in Figure 1.

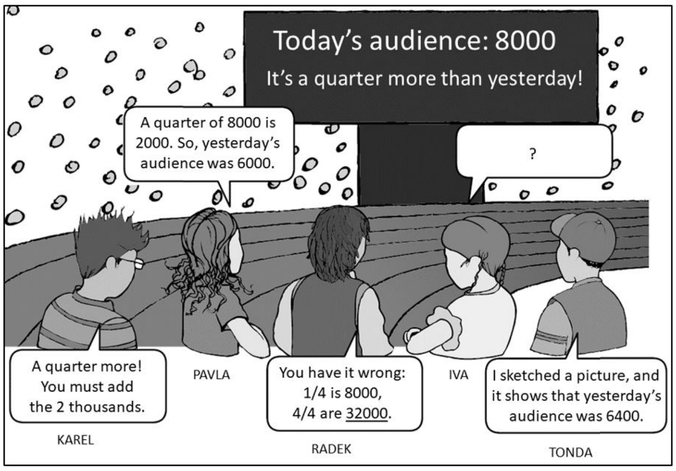


Figure 1: A Concept Cartoon related to the task T3*; (source of the template of children with empty bubbles: Dabell, Keogh and Naylor, 2008: 2.16)

The participants were asked to decide which children in the picture were right and which were wrong and to justify their decision. The form of the work with the Concept Cartoon was the same as with the test: individual and written. The task T3* has a similar structure and a similar difficulty as the task T3, see the attributes of the two tasks given in Table 1.
The method of how to use Concept Cartoons for diagnosing knowledge of future teachers and the particular Concept Cartoon from Figure 1 had been already tested previously (Samková and Tichá, 2017b; Samková, 2018a). This particular Concept Cartoon combines three bubbles containing procedures and results (Pavla, Karel, Radek), and a bubble introducing a result with a reference to a missing drawing that leads to the result (Tonda). The three bubbles with procedures and results are based on three most frequent incorrect solutions of the task T3*, and the fourth bubble without a procedure refers to a correct solution.

Data collection and data analysis

The study was performed in two separated consecutive stages: the first stage with the first group of participants, and the second stage with the second group of participants. From each of the participants, the data were collected at one time: first, the participant solved the written test with the word problem(s), and submitted it, and immediately after he/she worked on the Concept Cartoon.
For the first group of participants, the test served as a part of the course assessment, i.e. it took place after the topic of fractions was discussed at lectures and properly practised at course seminars. For the second group of participants, the test was an optional activity; the course with the topic of fractions and its assessment preceded my survey.
At the beginning of data analysis, I processed data from individual stages separately. When analysing data from the written test, I initially registered combinations of word problems that were successfully solved by individual participants (applies only to data from the first stage) and then monitored strategies that the participants used during the solution process. When analysing data from the Concept Cartoon, I initially registered combinations of bubbles that were chosen by individual participants as right, combinations of bubbles that were chosen as wrong, and strategies that the participants used in their justifications. Afterwards, I analysed mutual relations between data obtained via the word problem(s) and data obtained via the Concept Cartoon, and mutual relations between data obtained during the first and second stages.

Results

Written test – the first stage

Initial analysis of data related to written tests handled by the first group of participants showed that for all of the participants the success directly depended on the difficulty of the tasks:

- T4 was successfully solved only by participants who succeeded in T1, T2 and T3;
- T3 was successfully solved only by participants who succeeded in T1 and T2;
- T2 was successfully solved only by participants who succeeded in T1.

Such an arrangement allowed me to divide participants into five categories according to their success, and I labelled the categories by numbers corresponding to the most difficult tasks that the participants successfully solved: WT0 (no task solved), WT1 (only T1 solved), WT2 (only T1 and T2 solved), WT3 (only T1, T2 and T3 solved), WT4 (all tasks solved). There were 2 participants in WT0, 6 in WT1, 2 in WT2, 7 in WT3, and 6 in WT4.

1 In the sense of the primary school curriculum in the Czech Republic, which is the country of the first group of participants. *The framework of primary school mathematics* will be employed in this sense for the rest of the text.

In further analysis, I focused in detail on the task T3. The first stage participants in their solutions to T3 offered four different numbers as results and achieved these results by six different procedures. The results and samples of corresponding procedures are presented in Table 2.

80 [13]	75 [6]	240 [3]	300 [1]
60… 3/4 60 : 3 = 20… 1/4 20 · 4 = **80**… 4/4	60 : 4 = 15… 1/4 60 + 15 = **75**	1/4 = 60 4/4 = 60 · 4 = **240**	60 · 4 = 240 240 + 60 = **300**
	now… 4/4… 60 before… 5/4 60 : 4 = 15, 15 · 5 = **75**	1/4 from 60 = = 60 · 4 : 1 = **240**	

Table 2: Various results and various solution procedures to the task T3 given by the first stage participants, the column with the correct result is shaded; numbers of participants with a given result are indicated in square brackets

We can see that both the procedures leading to the result 75 proceeded from the incorrect identification of the whole; they were based on a similar misconception as in the Pavla's bubble. The first procedure leading to the result 240 might proceed from careless reading and understanding the text as "to a quarter" instead of "by a quarter"; a similar misconception as in the Radek's bubble. The second procedure leading to the result 240 combined two diverse misconceptions: an incorrect decision to calculate a quarter of 60 to get the result, and a calculation error consisting of reversing the order of division and multiplication when calculating the quarter of 60. The source of the decision to calculate a quarter of 60 is not clear, it might be a consequence of a strategy "take all numbers from the assignment, and do something with them" which sometimes appears among students (Samková and Tichá, 2015). The source of the calculation error probably lies in an unsuccessful effort to learn the calculation procedure by rote. The procedure leading to the result 300 might have a similar source as the first procedure of 240 – a response to a signal "before discount" causing the need for addition as the next step in the procedure. But the participant with the 300 result did not specify any fractions in the solution procedure, so that the source might also come from the "take all numbers" strategy mentioned above.

Concept Cartoon – the first stage

Since the Concept Cartoon was not compulsory and had no influence on the assessment of the course, seven of the first stage participants decided not to take part in this activity. There was no relation between their success in the written test and the decision not to take part in the Concept Cartoon part: each of the WT categories was represented among those who refused, by one or two participants. Due to the lack of data from these participants, I had to remove them from the study. So that only 16 participants remained for the first stage analysis involving the Concept Cartoon.

According to responses to the Concept Cartoon, the first stage participants might be divided into two categories: those who expressed the opinion that Tonda was right and the others wrong, and those who expressed the opinion that Pavla was right and the others wrong. All the opinions were justified by presenting a solution procedure that the participants considered as correct. Three of the solution procedures were also accompanied by illustrative pictures: one picture as a support for Pavla, and two pictures as a support for Tonda. Samples of solution procedures and illustrative pictures are shown in Table 3.

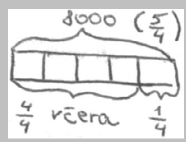
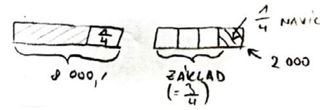
Tonda [9]	Pavla [7]
Only Tonda recognized that 8000 is a quarter more than the whole. The whole is 4/4, a quarter more is 5/4. 8000… 5/4 8000 : 5 = 1600… 1/4 8000 – 1600 = **6400**	Pavla is true. altogether… 8000 yesterday… a quarter less than 8000 : 4 = 2000 8000 – 2000 = **6000**
8000 : 5 = 1600 1600 · 4 = **6400** 	Tonda: Where is the picture? Incorrect answer! The picture should be this way:  8000 : 4 = 2000 came extra 2000 · 3 = **6000** yesterday

Table 3: Various responses to the Concept Cartoon given by the first stage participants, the column with correct responses is shaded; numbers of participants who agreed with a given child are indicated in square brackets; translation of texts in embedded pictures: včera = yesterday, základ = the whole, navíc = extra

Mutual relations – the first stage

According to combinations of results to the task T3 and opinions to the Concept Cartoon, the first stage participants might be divided into 7 categories, as shown in the diagram in Figure 2. Due to the similarities between the task T3 and the task behind the Concept Cartoon, some of the combinations might be labelled as corresponding, the others as non-corresponding. The corresponding combinations consisted either of both responses correct (80 & Tonda) or of both responses incorrect and based on a similar misconception (75 & Pavla). Such combinations accounted for half of the participants. The other half of the participants displayed non-corresponding combinations of responses: either both incorrect but based on different misconceptions (240 & Pavla, 300 & Pavla), or one correct and one incorrect (80 & Pavla, 75 & Tonda, 240 & Tonda). The most frequent non-corresponding combination was 80 & Pavla.

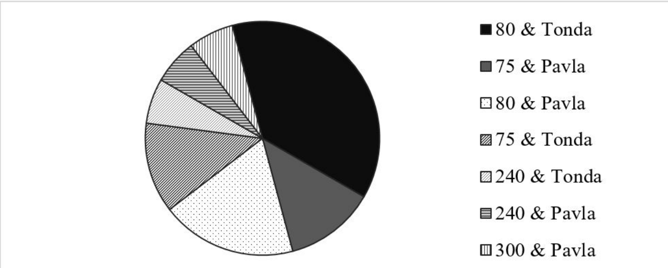


Figure 2: Combinations of responses to the word problem T3 and to the Concept Cartoon given by the first stage participants, corresponding combinations are colored, non-corresponding combinations are dotted or hatched, n=16, 2017 (source: own calculation)

Three of the non-corresponding combinations are noteworthy: 80 & Pavla, 75 & Tonda, and 240 & Tonda. Participants with a combination 80 & Pavla presented themselves successfully in the written test: they managed to solve the tasks T1, T2 and T3 (i.e. they belonged to the category WT3), some of them even solved the task T4 (category WT4). But responses to the Concept Cartoon showed a misconception about fractions: all of them incorrectly identified the whole in a task, presented incorrect solution procedures, and offered *justifications* for these incorrect procedures. Even the justifications did not warn them that something might not be right in their procedures.

Participants with combinations 75 & Tonda and 240 & Tonda were all weak in the written test: one of them did not succeed

in any of the test tasks (category WT0), the others successfully solved only the task T1 (category WT1). But with the Concept Cartoon, they all offered a correct solution procedure justifying the Tonda's bubble. Such an arrangement is surprising; the reason for the discrepancy might lie in the different format of the Concept Cartoon (e.g. in the fact that the numerical result of the correct solution appears inside one of the bubbles) or in the non-compulsory nature of the work with the Concept Cartoon or somewhere else; an exact determination would require more data.
The other combinations were more or less expected: good test solvers that responded correctly to the Concept Cartoon (80 & Tonda), and weak test solvers that responded incorrectly to the Concept Cartoon (75 & Pavla, 240 & Pavla, 300 & Pavla).

Written test – the second stage

The second stage participants in their solutions to T3 offered four different numbers as results. They achieved these results by seven different procedures within the framework of primary school mathematics, and by six different procedures outside the framework of primary school mathematics. Two participants failed to complete the task, two participants offered the correct result but without a procedure, and two participants offered just an incorrect result 75 without a procedure. The results with samples of solution procedures are presented in Tables 4 and 5.

80 [12]	75 [1]	240 [4]	180 [1]
3/4 = 60 60 : 3 = 20 60 + 20 = **80**	60 : 4 = 15 60 + 15 = **75**	1/4 = 60 4/4 = **240**	1/4… 60 3 · 60 = **180**
3/4… 60 crowns 1/4… 20 crowns 4/4… **80** crowns			
3/4… 60 3/4 + 1/4 = **80**			
1/4 of 80 = 20 80 – 20 = 60 **80**			

Table 4: Various results and various solution procedures to the task T3 given by the second stage participants within the framework of primary school mathematics, the column with the correct result is shaded; numbers of participants with a given result are indicated in square brackets

80 [4]	80 [14]	240 [2]
x – 1/4 x = 60 4x – x = 240 3x = 240 x = **80**	100 % – 25 % = 75 % 75 % = 60 75 : 60 = 100 : x 6000 : 75 = x x = **80**	x · 1/4 = 60 x = **240**
3/4 of the price… 60 crowns 4/4 = whole price… x x: 60 = 4/4 : 3/4 3/4 x = 60 · 4/4 3/4 x = 60 3x = 240 x = **80**	75 %… 60 crowns 100 %… x crowns x: 60 = 100 : 75 75 x = 6000 x = **80**	1/4 x = 60 x = **240**

Table 5: Various results and various solution procedures to the task T3 given by the second stage participants outside the framework of primary school mathematics, the columns with the correct result are shaded; numbers of participants with a given result and similar solution procedures are indicated in square brackets

Table 4 contains procedures within the framework of primary school mathematics, and Table 5 contains procedures outside the framework of primary school mathematics (i.e. procedures using equations with one unknown, ratios, percentages). The first shaded column in Table 5 contains correct procedures that do not employ percentages, and the second shaded column contains correct procedures that employ percentages. In Table 4, the last procedure in the first column is interesting: here the participant did not solve the task but guessed or estimated its result, and then verified it.

Concept Cartoon – the second stage

According to responses to the Concept Cartoon, the second stage participants might be divided into the same two categories as in the first stage: those who agreed only with Tonda, and those who agreed only with Pavla. There were five participants who did not offer any justification for their decisions: one of them agreed with Tonda, and four agreed with Pavla. Three of the participants offered justifications via illustrative pictures, all of them as support for Pavla. The other decisions were justified by presenting a solution procedure that the participants considered as correct. Samples of solution procedures and samples of illustrative pictures are shown in Tables 6 and 7: Table 6 contains procedures within the framework of primary school mathematics, and Table 7 contains procedures outside the framework of primary school mathematics. The fourth solution procedure in the second column of Table 6 is unique: it combines a mistake in a fractional representation and a calculation mistake, and its fractional representation does not relate to any other in collected data.

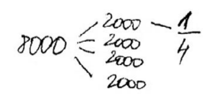
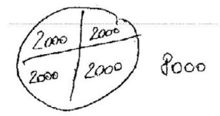
Tonda [7]	Pavla [24]
Tonda calculated correctly: 6400 was yesterday, a quarter of it is 1600, and 6400 + 1600 = 8000.	Pavla is right: 1/4 of 8000 = 2000 thus 8000 – 2000 = **6000** viewers, because there were less viewers yesterday than today
Tonda is right. When we divide 6400 to quarters, we get 1600. And when we add 1600 to 6400, we get 8000. Pavla is not right. If yesterday came 6000 people, then a quarter would be 1500, and the audience today would be only 7500.	1/4 of 8000 is 2000 3 · 2000 = **6000**
	4/4… 8000 1/4… 2000 4/4 – 1/4 = 8000 – 2000 = **6000**
Tonda is right. That yesterday's quarter is 1600.	8000 are 3/4, which means that 8000: 4 · 3 = **6000** people yesterday
	
	

Table 6: Various responses to the Concept Cartoon given by the second stage participants within the framework of primary school mathematics, the column with correct responses is shaded; numbers of participants who agreed with a given child and offered similar solution procedures or illustrative pictures are indicated in square brackets

Tonda [5]	Pavla [3]
125 %… 8000 100 %… x 8000 · 100 = 125 x **6400** = x	8000… 100 % x… 75 % x: 8000 = 75: 100 100 x = 8000 · 75 100 x = 600 000 x = **6000**
x · 5/4 = 8000 x = 32000: 5 x = **6400**	

Table 7: Various responses to the Concept Cartoon given by the second stage participants outside the framework of primary school mathematics, the column with correct responses is shaded; numbers of participants who agreed with a given child and offered similar solution procedures are indicated in square brackets

Mutual relations – the second stage

According to combinations of results to the task T3 and opinions to the Concept Cartoon, the second stage participants might be divided into 8 categories, as is shown in the diagram in Figure 3. As in the first stage, two of the combinations might be labelled as corresponding, and the others as non-corresponding. This time, the corresponding combinations accounted only for less than a third of the second stage participants, and almost half of the second stage participants accounted for the most frequent non-corresponding combination 80 & Pavla (these participants solved the word problem correctly but the Concept Cartoon incorrectly).

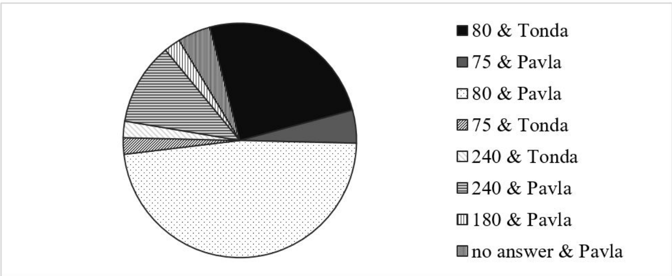


Figure 3: Combinations of responses to the word problem T3 and to the Concept Cartoon given by the second stage participants; corresponding combinations are colored, non-corresponding combinations are dotted or hatched, n=44, 2018 (source: own calculation)

The first stage vs the second stage

Results from the first and second stages differ in two noticeable ways. Firstly, in solution procedures that the stage participants as a whole used to solve the task T3, and secondly, in correspondence between solution procedures that individual stage participants used to solve the word problem T3 and the corresponding Concept Cartoon problem. The first stage participants were less successful than the second stage participants in solving the task T3, and most of the second stage correct solution procedures belonged outside the primary school mathematics. In the first stage, the most frequent correct solution procedure was the only correct one that appeared in data, but in the second stage, there were four different correct procedures.

In the first stage, the corresponding combinations of procedures to the word problem and the Concept Cartoon appeared in half of the cases, in the second stage only in less than a third. The prevailing combination in the first stage was the corresponding combination 80 & Tonda, while the prevailing combination in the second stage was the non-corresponding combination 80 & Pavla.

All the above differences are probably consequences of different contexts (educational as well as organizational) in which the first and second stages of the survey took place: the curricula are not the same in the two countries, nor the course of the university training for future primary school teachers, also the organization of data collection varied in range of tasks assigned to the participants and (non)existence of additional requirements on solution methods.

Regardless of the context differences, the key finding is the same for both stages: there appeared a substantial group of successful solvers of the word problem T3 that solved the corresponding problem in the Concept Cartoon form incorrectly. In the first stage, this group accounts for one third of all the successful solvers of T3, and in the second stage for almost two thirds.

Discussion

The results of this study enriched the puzzle on *"How can we meaningfully employ Concept Cartoons in future teacher education"* by another piece of knowledge. They give a positive answer to the research question *"Can Concept Cartoons provide us with information on mathematics content knowledge that might not be obtained through word problems in standard written tests?"*

In contrast with standard written tests, Concept Cartoons may reveal participants who look like good test solvers capable to solve word problems of any difficulty, but their capability is just an illusion. For instance, the participants of the first stage of my study who belonged to the WT4 & 80 & Pavla combination of categories: they might be considered as excellently mastering the topic of fractions on the basis of the written test, but with the Concept Cartoon they displayed a fundamental misconception – incorrect determination of the whole.

There are two different mechanisms that allow Concept Cartoons to uncover the written test illusion: (i) Concept Cartoons offer several alternative viewpoints on the pictured situation, so that they may break the stereotype of "favourite" or "comfortable" solution procedures that the solvers learned for the purpose of the written test, and may tempt the solvers to incline to some of the other procedures; (ii) when working with Concept Cartoons, the solvers are asked to provide justifications of their agree/disagree decisions, and so they expose their reasoning on the explored topic outside the common framework of problem solving.

These findings are important in light of the fact that the participants of the referred study were future teachers. Considering the way how Concept Cartoons make the respondents to reason not only in the framework of their "favourite" or "comfortable" interpretation of the topic but also in the framework of other interpretations, we may understand this tool as an artificially designed representation of school practice (Samková, 2018a), as a result of the process that Grossman et al. (2009) call a decomposition of practice into constituent parts. With such representations, we can engage future teachers in discussions about various aspects of teaching (mathematics content, classroom communication, etc.), and the representations may serve as mediating tools between teaching practice and future teacher education (Herbst and Chazan, 2011). In that sense, the representations may also indirectly promote the development of noticing mathematics specific phenomena and knowledge-based reasoning (van Es and Sherin, 2002; Vondrová and Žalská, 2015). As Star and Strickland (2008: 123) point out appositely, "preservice teachers have previously observed countless hours of mathematics instruction" but "their observations have been as *learners* of mathematics, not as *teachers* of mathematics". Findings of this study give a clear illustration of the quote.

From the perspective of the topic of fractions, the findings confirmed the difficulty of the topic for future primary school teachers that was reported e.g. by Cramer and Lesh (1988), Ma (1999), Depaepe et al. (2015): almost a third of the participants of the referred study did not solve correctly a word problem based on a fractional change of the whole (T3), as they did not grasp the task properly and/or did not identified properly the whole in the task. This happened also with a similar word problem (T3*) which was assigned in the Concept Cartoon form – in that case, more than two thirds of participants agreed with a bubble that identified incorrectly the whole in the task. Even the correct result that numerically appeared inside one of the other bubbles did not help. Unfortunately, the large number of unsuccessful solvers of word problems with a fractional change is not exceptional, a task similar to T3* appeared in 2015 in the Czech

Republic as a word problem in the state matriculation exam, with only 33 % of the students that solved the task correctly (Samková, 2018a). Some factors that cause the difficulty of the tasks with fractional changes have been already mentioned in the Introduction section, the others might relate to the fact that this kind of word problems rarely appears in explanatory parts of textbooks and learning materials on fractions, it is not even included in summarizing books on fractions or misconceptions (Lamon, 1999; Ryan and Williams, 2011).

From the perspective of solution strategies that the participants used when they solved the tasks, the findings meet the results of previous research where similar tasks were used (Lamon, 1999; Tichá and Macháčková, 2006; Samková, 2018a): typical correct solution procedures as well as common misconceptions appeared in solutions, some of them accompanied by visualizations.

A comparison of solution strategies and results related to the two similar tasks (T3, T3*) that were assigned in two different forms (word problem, Concept Cartoon) illustrates how diverse information can be provided by word problems and Concept Cartoons: only a third of the participants displayed corresponding responses to the two forms of problems: either both responses correct and based on a similar strategy (80 & Tonda), or both responses incorrect and based on a similar misconception (75 & Pavla). The remaining two thirds of participants responded correctly to one of the forms and incorrectly to the other (e.g. 75 & Tonda), or responded incorrectly in both cases but the responses were based on different misconceptions (e.g. 240 & Pavla). This finding is in line with conclusions of Novotná and Vondrová (2017) about the impact that the context of a task might have on solving strategies.

The weak point of the referred study consists in the impossibility to generalize the results. On the other side, I included as participants all future primary school teachers who came to the two compulsory lessons where data were collected – in that sense the study is representative.

Conclusion

This contribution investigated opportunities that an educational tool called Concept Cartoons could offer in future teachers' education, namely in comparison with word problems in standard written tests. From the perspective of mathematics content, it focused on the topic of fractions which again proved its difficulty for future primary school teachers.

The study confirmed the efficiency of using Concept Cartoons in future primary school teachers' education, since they may provide us with information on misconceptions that might not be obtained through standard written tests. I conducted the study with two groups of future primary school teachers from two neighbouring countries, in two diverse educational and organizational contexts. Regardless of the context differences, the key finding on the efficiency of Concept Cartoons in future primary school teachers education is the same for both stages.

References

Behr, M., Lesh, R., Post, T. and Silver, E. (1983) 'Rational number concepts', in *Acquisition of mathematics concepts and processes*, New York: Academic Press, pp. 91-125.

Cramer, K. and Lesh, R. (1988) 'Rational number knowledge of preservice elementary education teachers', in *Proceedings of PME 88*, DeKalb, Il: PME, pp. 425-431.

Dabell, J., Keogh, B. and Naylor, S. (2008) *Concept Cartoons in Mathematics Education*, Sandbach: Millgate House Education.

Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G., Verschaffel, L. and Van Dooren, W. (2015) 'Teachers' content and pedagogical content knowledge on rational numbers: a comparison of prospective elementary and lower secondary school teachers', *Teaching and Teacher Education*, vol. 47, pp. 82-92. http://dx.doi.org/10.1016/j.tate.2014.12.009

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shanan, E. and Williamson, P. (2009) 'Teaching practice: A cross-professional perspective', *Teachers College Record*, vol. 111, no. 9, pp. 2055-2100.

Herbst, P. and Chazan, D. (2011) 'On creating and using representation of mathematics teaching in research and teacher development', *ZDM Mathematics Education*, vol. 43, pp. 1-5. http://doi.org/10.1007/s11858-011-0306-9

Hošpesová, A., Tichá, M. (2015) 'Problem posing in primary school teacher training', in Singer, F.M., Ellerton, N.F. and Cai, J. (ed.) *Mathematical problem posing*, New York: Springer, pp. 433-447. http://doi.org/10.1007/978-1-4614-6258-3_21

Ingram, J., Pitt, A. and Baldry, F. (2015) 'Handling errors as they arise in whole-class interactions', *Research in Mathematics Education*, vol. 17, pp. 183-197. http://doi.org/10.1080/14794802.2015.1098562

Kazemi, E. and Stipek, D. (2001) 'Promoting conceptual thinking in four upper-elementary mathematics classrooms', *Elementary School Journal*, vol. 102, pp. 59-80. http://doi.org/10.1086/499693

Lamon, S.J. (1999) *Teaching fractions and ratios for understanding,* Mahwah, NJ: LEA.

Ma, L. (1999) *Knowing and teaching elementary mathematics,* Mahwah, NJ: Erlbaum.

Národní ústav pro vzdělávání (NÚV), (2015) *Rámcový vzdělávací program pro základní vzdělávání*, Praha: MŠMT, [Online], Available: http://www.msmt.cz/file/37052/ [5 Sep 2018].

Nesher, P. (1987) 'Towards an instructional theory: the role of student's misconceptions', *For the Learning of Mathematics*, vol. 7, no. 3, pp. 33-40.

Novotná, J. and Vondrová, N. (2017) 'Pupils' strategies for missing value proportional problems', in *Proceedings of ERIE 2017,* Prague, pp. 279-286, [Online], Available: http://erie.v2.czu.cz/en/r-13628-proceedings-2017 [5 Sep 2018].

Pavlovičová, G. and Švecová, V. (2017) 'Some misconceptions in understanding of fractions', in *Proceedings of EDULEARN17 Conference*, Barcelona, pp. 5463-5470. http://doi.org/10.21125/edulearn.2017.2246

Ryan, J. and Williams, J. (2011) *Children's mathematics 4-15. Learning from errors and misconceptions,* Berkshire: Open University Press.

Samková, L. (2018a) 'Concept Cartoons as a representation of practice', in *Mathematics Teachers Engaging with Representations of Practice. ICME-13 Monograph*, New York: Springer. http://doi.org/10.1007/978-3-319-70594-1_5

Samková, L. (2018b) 'Written tests vs Concept Cartoons: the case of future teachers and fractions', in *Proceedings of ERIE 2018,* Prague, pp. 314-321, [Online], Available: http://erie.v2.czu.cz/en/r-13780-proceedings-2018 [5 Sep 2018].

Samková, L. and Tichá, M. (2015) 'Investigating future primary teachers' grasping of situations related to unequal partition word problems', *Quaderni di Ricerca in Didattica (Mathematics)*, vol. 25, no. 2, pp. 295-303.

Samková, L. and Tichá, M. (2016a) 'Developing open approach to mathematics in future primary school teachers', in *Proceedings of ERIE 2016,* Prague, pp. 494-501, [Online], Available: http://erie.v2.czu.cz/en/r-13629-proceedings-2016 [5 Sep 2018].

Samková, L. and Tichá, M. (2016b) 'On the way to develop open approach to mathematics in future primary school teachers', *ERIES Journal*, vol. 9, no. 2, pp. 37-44. http://doi.org/10.7160/eriesj.2016.090202

Samková, L. and Tichá, M. (2017a) 'Observing how future primary school teachers reason about fractions', in *Proceedings of ERIE 2017,* Prague, pp. 363-371, [Online], Available: http://erie.v2.czu.cz/en/r-13628-proceedings-2017 [5 Sep 2018].

Samková, L. and Tichá, M. (2017b) 'On the way to observe how future primary school teachers reason about fractions', *ERIES Journal*, vol. 10, no. 4, pp. 93-100. http://doi.org/10.7160/eriesj.2017.100401

Schleppenbach, M., Flevares, L.M., Sims, L.M. and Perry, M. (2007) 'Teachers' responses to student mistakes in Chinese and U.S. mathematics classrooms', *Elementary School Journal*, vol. 108, pp. 131-147. http://doi.org/10.1086/525551

Singer, F.M., Ellerton, N.F. and Cai, J. (2015) *Mathematical problem posing*, New York: Springer. http://doi.org/10.1007/978-1-4614-6258-3

Star, J.R. and Strickland, S.K. (2008) 'Learning to observe: using video to improve preservice mathematics teachers' ability to notice', *Journal of Mathematics Teacher Education*, vol. 11, pp. 107-125. http://doi.org/10.1007/s10857-007-9063-7

Steffe, L. and Olive, J. (2010) *Children's fractional knowledge,* New York: Springer. http://doi.org/10.1007/978-1-4614-6258-3

Švecová, V., Pavlovičová, G., Rybanský, Ľ. and Klimentová, L. (2017) *Reifikácia zlomkov vo vzťahu k osobnej potrebe štruktúry,* Praha: Wolters Kluwer.

Tichá, M. and Macháčková, J. (2006) *Rozvoj pojmu zlomek ve vyučování matematice*, Praha: JČMF.

Uličná, K. (2017) 'Professional vision: focus on knowledge-based reasoning', in *Proceedings of ERIE 2017,* Prague, pp. 482-489, [Online], Available: http://erie.v2.czu.cz/en/r-13628-proceedings-2017 [5 Sep 2018].

van Es, E.A. and Sherin, M.G. (2002) 'Learning to notice: scaffolding new teachers' interpretations of classroom interactions', *Journal of Technology and Teacher Education*, vol. 10, pp. 571-596.

Vondrová, N. and Žalská, J. (2015) 'Ability to notice mathematics specific phenomena: what exactly do student teachers attend to?', *Orbis Scholae*, vol. 9, no. 2, pp. 77-101. http://doi.org/10.14712/23363177.2015.81

MATHEMATICAL PROBLEM-SOLVING STRATEGIES AMONG STUDENT TEACHERS

Melanie Guzman Gurat

School of Teacher Education and Humanities / School of Graduate Studies, University Research Center, Saint Mary's University, Philippines, melanie.gurat@yahoo.com

Highlights

- *Problem-solving strategies among student teachers are cognitive, metacognitive and other strategies*
- *Results indicate significant influence of the strategies on academic performance of the student teachers*

Abstract

The main purpose of the study is to understand the mathematical problem-solving strategies among student teachers. This study used both quantitative and qualitative type of research. Aside from the semi-structured interviews, data were gathered through participant's actual mathematical problem-solving outputs and the videotaped interviews. Findings revealed that the problem-solving strategies among student teachers in the Problem-Solving subject are cognitive, metacognitive and other strategies. The cognitive strategies used by the student teachers are rehearsal, elaboration, and organization. The metacognitive strategies are critical thinking and self-regulation. Other strategies are overlapping the cognitive and metacognitive strategies. These are prediction/orientation, planning, monitoring, and evaluating. The findings also suggest significant influence of the strategies on the academic performance of the student teachers.

Article type

Full research paper

Article history

Received: September 14, 2017

Received in revised form: August 29, 2018

Accepted: August 29, 2018

Available on-line: September 30, 2018

Keywords

Cognitive, critical thinking, elaboration, metacognitive, organization, rehearsal, self-regulation

Guzman Gurat M. (2018) "Mathematical problem-solving strategies among student teachers", *Journal on Efficiency and Responsibility in Education and Science, Vol. 11, No. 3*, pp. 53-64, online ISSN 1803-1617, printed ISSN 2336-2375, doi: 10.7160/eriesj.2018.110302.

Introduction

Problem-solving has a special importance in the study of mathematics (Wilson, Fernandez and Hadaway, 2011). The main goal in teaching mathematical problem-solving is for the students to develop a generic ability in solving real-life problems and to apply mathematics in real life situations. It can also be used, as a teaching method, for a deeper understanding of concepts. Successful mathematical problem-solving depends upon many factors and skills with different characteristics. One of the main difficulties in learning problem-solving is the fact that many skills are needed for a learner to be an effective problem solver. Also, these factors and skills make the teaching of problem-solving one of the most complex topics to teach (Dendane, 2009). Mathematics is used to quantify numerically and spatially natural as well as man-made situations. It is used to solve problems and it has helped in making social, economic and technological advances (Dendane, 2009).

Learning facts and contents in mathematics are important but these are not enough. Students should learn how to use these facts to develop their thinking skills in solving problems. Special attention for the development of problem-solving ability has been accepted by mathematics educators (Stanic and Kilpatrick, 1989) and genuine mathematical problem-solving is one of the most important components in any mathematics program or curriculum (Stacey, 2005; Halmos, 1980; Cockcroft, 1982). Mathematical problem-solving may help students to improve and develop the standard ability to solve real-life problems, (Reys et al. 2001), to develop critical thinking skills and reasoning, to gain deep understanding of concepts (Schoenfeld, 1992; Schoen and Charles, 2003) and to work in groups, cooperate with and interact with each other (Dendane, 2009). Specifically, it may also improve eagerness of an individual to try to analyze mathematical problems and to improve their determination and self-concepts with respect to the abilities to solve problems; make the individual aware of the problem-solving strategies, value of approaching problems in an orderly manner and that many problems can be solved in more than one way and; improve individuals' abilities to select appropriate solution strategies, capacity to implement solution strategies accurately and abilities to get a correct answers to problems (Hoon, Kee, and Singh, 2013).

A heuristic is a mathematical problem-solving strategy formulated in a free-of-context manner and done systematically (Koichu, Berman and Moore, 2004). Moreover, a heuristic approach can encourage connection of mathematical thoughts by examining special cases, drawing a diagram, specializing the solution, generalizing the solution (Hoon, Kee, and Singh, 2013). It is associated with non-routine mathematical problems such as looking backward or thinking forward (Koichu, Berman and Moore, 2004). Several studies were conducted to improve students' skills in solving mathematics problems. Hoon, Kee, Singh (2013) investigated students' response in applying heuristics approach in solving mathematical tasks, and their abilities in applying the heuristics approach. Reiss and Renkl (2002) proposed the use of heuristic worked-out examples in proving. They suggested that this should be integrated into mathematics classroom frequently so that students will learn to extract needed information in the problems. Novotná (2014) aimed to improve the pupils' culture of problem-solving through dealing with strategies such as analogy, guess-check-revise, problem reformulation, solution drawing, systematic experimentation, way back and use of graphs of functions With the studies showing how strategies can improve mathematics problem solving, Koichu, Berman, and Moore (2004) aimed to promote heuristic literacy in a regular mathematics classroom.

Moreover, Dewey's (1933) "How we think", Polya's (1988) problem-solving methods and the stages of Krulik and

Rudnick's (1978) in solving were some of the theoretical bases in conducting this study (cited by Carson, 2007). These theories explained problem-solving as strategies in solving. Dewey's (1933) steps are confronting the problem, diagnosing or defining the problem, inventorying several solutions, conjecturing consequences of solutions and testing the consequences. On the other hand, Polya's (1988) stages consist of understanding the problem, devising a plan, carrying out the plan and looking back. Similarly, Krulik and Rudnick's (1978) procedures are reading, exploring, selecting a strategy, solving and reviewing and extending. These theories serve as a guide to a researcher to work on particular strategies performed by the student teachers while dealing with the mathematical problem-solving task. In this study, problem-solving refers to the common situational problems in mathematics in a form of problem set or worded problems. The problems are composed of items in arithmetic and algebra, trigonometry, geometry, sets, probability, number theory and puzzle problem/logic.
Hence, with the main goal of mathematics education to improve students' problem-solving skills in mathematics particularly to the student teachers who will be future mathematics educators, this study aimed to understand the mathematical problem-solving strategies among student teachers. This study can be used as a basis for the tertiary mathematics educators to determine different methods or interventions to improve the problem-solving skills of the future teachers so that they will be equipped with enough skills in teaching mathematics for their future students. It can also serve as a realization for them to grow more sensitive to different strategies and to realize that there are more strategies in solving problems in mathematics.

Materials and Methods

The study was qualitative. Semi-structured interviews, participant's actual mathematical problem-solving outputs, Filled-up Mathematics Motivated Strategies Learning Questionnaires (MMSLQ) by Liu and Lin (2010) (See Appendix A) and videotaped interviews were used to triangulate the gathered data. Techniques and analytical tools by Strauss and Corbin (1998) and the constant comparison method by Glaser and Strauss (1967) were used. The data used in the study was the initial process conducted to determine metacognitive strategy knowledge in the study of Gurat and Medula (2016). The identified strategies were used by Gurat and Medula in constructing a framework of metacognitive strategy knowledge in solving math problems. The participants of the study were the student teachers who were currently enrolled in Problem-Solving subject during the summer 2011 term. Student teachers are the senior college students of Saint Mary's University officially enrolled in Problem-Solving subject. The class is composed of 23 students, 19 of which are Bachelor of Elementary Education major in General Elementary Education (BEED – GEE), 4 Bachelor of Secondary Education major in Mathematics (BSED Math) and 1 Bachelor of Elementary Education major in General Science (BEED General Science), 19 females and 4 males. Out of 23 students, only 12 BEED – GEE students were willing to be interviewed. Out of 19 females, there are only 10 females interviewed and out of 4 males, there are only 2 males interviewed. The scores of the student teachers in the Mathematics problem set or their grades in Problem-Solving subject were not used as a criterion for identifying the respondents to be interviewed. Table 1 shows the course and year, gender, grade in Problem-Solving subject and scores of interviewed and not interviewed student teachers in the given problem set and their grades in Problem-Solving subject.

Name	Course & Year	Gender	Grade in Problem-solving	Score (out of 22 points)
Interviewed				
Ana	BEED 4	F	80	5
Barbara	BEED 4	F	83	2
Carding	BEED 4	M	81	1
Clara	BEED 4	F	85	3
Ester	BEED 4	F	86	6
Grasya	BEED 4	F	85	4
Helen	BEED 4	F	89	8
Inday	BEED 4	F	89	4
Isagani	BEED 4	M	95	9
Maria	BEED 4	F	84	3
Selya	BEED 4	F	86	4
Soledad	BEED 4	F	89	5
Not Interviewed				
Delya	BEED 4	F	85	6
Elyas	BEED 4	M	77	5
Esteban	BEED 4	M	86	3
Fatima	BSED 4	F	88	7
Julieta	BEED 4	F	87	5
Katrina	BSED 4	F	97	8
Lusing	BSED 4	F	97	8
Nena	BESD 3	F	94	12
Perla	BEED 4	F	82	7
Tina	BEED 4	F	87	6
Wilma	BEED 3	F	inc	4

Table 1: Course and year, gender, grade in Problem-Solving subject and scores of interviewed and not interviewed pre-service teacher education students

The instruments used in the study underwent tool validation and pilot testing. Revisions on the instruments were done before the student teachers were given the problem set (see Appendix B).
The data gathering procedure started upon the approval to conduct this study. The student teachers answered the given set of problem-solving and the Mathematics Motivated Strategies Learning Questionnaires. The outputs of the students in the problem set and the result on the MMSLQ questionnaires were analyzed to construct the guide questions for the interview (see Appendix C). Semi-structured interviews were conducted at Roger Tjolle Building, second floor conference room of Saint Mary's University. The interviews were recorded and videotaped to validate/support interview responses. The interviews were transcribed and the transcriptions were analyzed through Strauss and Corbin coding process. In this stage, microanalysis was done which includes both open coding and axial coding. Then, related concepts were grouped together using axial coding. The categories formed were analyzed word-for-word, line-by-line and sentence-by-sentence. Tables 2 and 3 show the sample excerpts from the open coding and axial coding respectively. Based on the concepts generated from the raw data, categories and subcategories were formed by constant comparison. Selective coding was also done to identify the themes formed from the axial coding. Finally, the result of the study was reported to student teacher for verification purposes.

English Translations	Behaviors/ Type of strategies Sub categories/Others
Mathematical problem solving is about applying the formula and it is a systematic process. So meaning it is a step-by-step process to get the correct answer	Systematic Approach (Monitoring) Relate math to formulas
I use the formulas (if familiar with the problem)	Use of formulas
If not, I analyze first the problem before solving for the right answer.	Analysis of information
I read and understand it first then identify the needed data	Read, analyze, solve method of solving
I set aside the problem then I will ask for help from my classmate or I'll search for problems that can be used to relate to them	-Categorize easy-hard question by skipping items that are difficult to answer (Organization) -Looking Back at the problem -Social -relate to other problem (critical thinking) -Speculation
I leave it ma'am, I do guessing but I feel it's wrong If I really don't know it then no more	-guessing/trial and Error -Explore/discover
It's like it's already in my mind like when we have a lesson that I understood it so I can imagine it.	-recall lesson(rehearsal) -analysis of information
During elementary, basic math was taught to us. Read what is the problem, and then first you analyze it and find the given and then identify the specific question asked in the problem	-systematic approach (monitoring) -recall past lesson (rehearsal) -analysis of information
I'm thinking about it, how I could answer the given question	-asking question (Elaboration) -constructing meaning and developing an interpretation
I read it first then I find ways to solve what is being asked in the problem	-exploring/discover -critical thinking
Sometimes if I really don't know, I read it again and again	-reading repeatedly (rehearsal, prediction/ orientation) -Constructing meaning and developing an interpretation
Hhmmm the questions seem like something given that…aaaayyyy I will think how to solve it	-explore/discover -asking self (elaboration)

Table 2: Extract from open coding of interview transcripts

What	When does the category occur	Why does the category occur	How does the category occur	Consequences
Constructing meaning and developing the interpretation	during the first phase of the problem solving	primary encounter and sense-making	-listing -making drawing, illustrations, tables, chart -reading the problem again and again	To understand the problem
Analyzing information			-selecting relevant information -relating it to a certain mathematical field	To Analyze the problem
Looking back on the problem			- recalling similar problems -assessing the degree of difficulty	To Analyze the problem
Exploring/ Discovering	During the second phase of the problem solving	Planning what to do	-Using trial and error -visualizing the situation -establishing a connection among part of the problem -analyzing the problem part by part	Preparatory to design a plan. For better analysis
Speculating			- relating it to real life situation -relating to a similar problem encountered before.	Preparatory to design a plan. For better analysis
Reflecting on the discovery and speculation			-decision making whether feasible or not	Reflecting

Table 3: Extract from axial coding of interview transcripts

Results

Based on the transcriptions of the interviews, filled-up Mathematics Motivated Strategies Learning Questionnaires (MMSLQ) and scanned outputs in their actual problem-solving tasks, the strategies identified were cognitive, metacognitive and other strategies.

Cognitive Strategies

Three kinds of cognitive strategies were identified in this study. These include rehearsal, elaboration, and organization.

Rehearsal

Rehearsal is one of the cognitive strategies used by the student teachers in Summer 2011 Problem-Solving subject. Rehearsal is shown through re-reading the problem, solving problems repeatedly and recalling past lessons.

In addition, Table 4 shows the frequency and percent distribution of cognitive strategy of rehearsal used by the student teachers in solving mathematical problem-solving. The table reveals that the student teachers make use of the cognitive strategy of rehearsal since they responded that they sometimes or even always used their cognitive strategies. Only one respondent said that s/he repeatedly practice similar question types.

Cognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I analyze the problem again and again.	0	0	0	0	7	30.44	8	34.78	8	34.78
I repeatedly practice similar question types.	1	4.35	2	8.70	11	47.83	7	30.44	2	8.70
I study the class notes and textbook again and again.	0	0	5	21.74	11	47.83	5	21.74	2	8.70
I memorize the important and key math formula to remind me of the important part of my math class	0	0	4	17.39	6	26.09	9	39.13	4	17.39
I do not forget problem-solving steps	0	0	6	26.09	12	52.17	4	17.39	1	4.35

Table 4: Frequency and percent distribution of the cognitive strategies of rehearsal used by the student teachers in solving mathematical problems

Elaboration

Elaboration was used by the student teachers in solving mathematical problems. This strategy was shown through underlining and selecting important details such as words and given in the problem and asking own self-questions related to solving. Table 5 shows that student teachers used elaboration in solving mathematical problems. If not sometimes true about half of the time or frequently, some also responded that they use it always.

Cognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I ask questions to myself to make sure that I understand the math materials content	0	0	4	17.39	9	39.13	7	30.43	3	13.04
I link the class notes to textbook examples to improve my understanding.	0	0	3	13.04	9	39.13	10	43.48	1	4.35
I combine my own known knowledge with the learning materials.	0	0	2	8.70	10	43.48	9	39.13	2	8.70
I do my best to link relative portions of math and other subjects.	1	4.35	2	8.70	12	52.17	8	34.78	0	0
I will find out any sample in daily life to link with math materials.	0	0	4	17.39	15	65.22	4	17.39	0	0

Table 5: Frequency and percent distribution of the cognitive strategies of elaboration used by the student teachers in solving a mathematical problem

Organization

The organization was shown by the student teachers by making connections between parts of the problem, making a drawing of the problem statement, and breaking down the problem into pieces, making simple charts/tables to better organize what is asked in the problem.

Problem solvers make connections between the parts of the problem in order to decide which of the following given are needed. They claim that if a solver did not get or understand the connection between parts of the problem he may fail to get the correct answer, especially that some problems have missing numbers needed to be solved first before solving what is really asked in the problem. It is also through making connections between parts of the problem that a problem solver may decide what strategy/formula/method/steps should fit the question.

Furthermore, Figure 1 and 2 show the sample output revealing that student teachers make drawings.

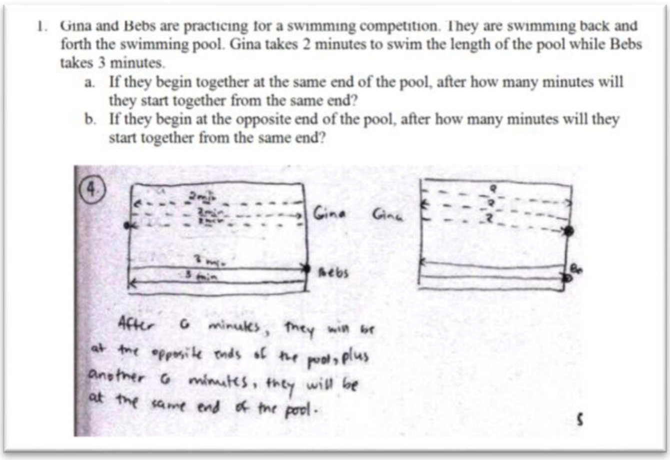


Figure 1: Drawing of Katrina

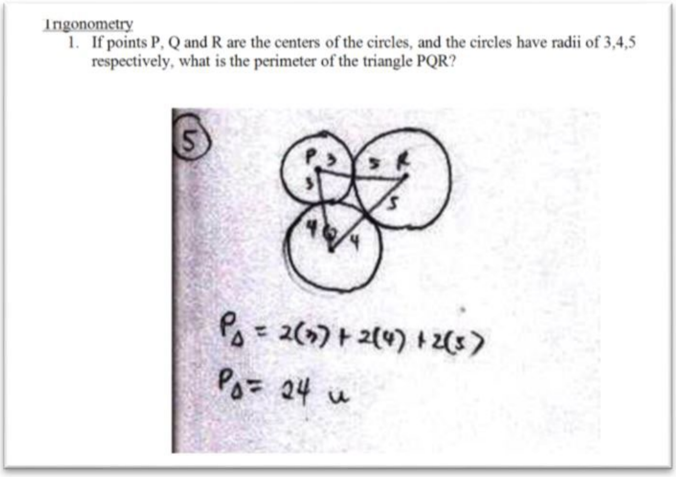


Figure 2: Drawing of Lusing

Making a drawing of the problem statement is evident especially if the given problem requires illustration before one can solve it. Examples are shown in Figures 1 and 2.

The organization can also be shown through making table.

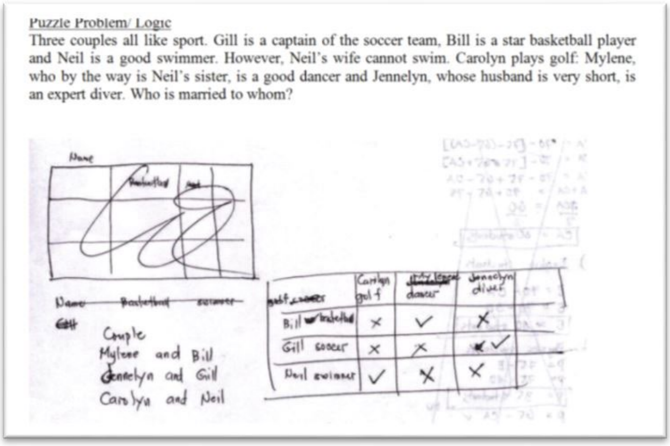


Figure 3: Table drawn by Helen

Figure 3 does not just reveal that student teachers make tables but it also shows the use of rehearsal. Helen draws table but disregarded it maybe because she repeats reading the problem.

Though some respondents answered "no" when asked if they break down the problems into pieces, make simple charts/ tables to better organized what is asked in the problem, this is contradictory to their output revealing that the student teachers actually make charts/tables in answering a problem. One reason might be because the problem requires a solver to do so even if it is not written there that they must make table/charts. Thus, this also reveals that a solver may or may not be aware of their cognitive strategies.

In addition, Table 6 shows the frequency and percent distribution of cognitive strategy of organization used by the student teachers in solving mathematical problems. Only two respondents responded that they did not underline important words in the word problem but for the rest of the items, the table shows that they use the other strategies sometimes or even always. Thus, this shows that the student teachers used a cognitive strategy of the organization in solving.

Cognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I mark-up the important lines for concepts organization.	0	0	4	17.39	7	30.43	7	30.43	5	21.74
I underline important words in the word problem	2	8.70	1	4.35	9	39.13	7	30.43	4	17.39
I select relevant numbers/data to solve the problem	0	0	2	8.70	6	26.09	10	43.48	5	21.74
I adhere to the plan systematically	0	0	2	8.70	10	43.48	10	43.48	1	4.35
I take time to design an action plan before actually calculating	0	0	4	17.39	6	26.09	9	39.13	4	17.39
I read through the class notes and textbook and find out the most important parts.	0	0	4	17.39	6	26.09	9	39.13	4	17.39
I read through the class notes and mark up the important parts.	0	0	4	17.39	9	39.13	7	30.43	3	13.04
I categorize the easy-hard type questions of every exam.	0	0	4	17.39	7	30.43	9	39.13	3	13.04
I orderly take note of problem-solving steps	0	0	4	17.39	11	47.83	5	21.74	3	13.04
I make simple charts and tables to help me in organizing my math class materials.	0	0	1	4.35	11	47.83	9	39.13	2	8.70
I select the calculations that will be needed to solve the problem and estimating a possible outcome	0	0	3	13.04	10	43.48	3	13.04	7	30.43
I act according to the plan	0	0	5	21.74	13	56.52	5	21.74	0	0
I follow the sequences of problem-solving steps orderly	0	0	5	21.74	12	52.17	5	21.74	1	4.35
I go over the formula and important concepts by myself.	0	0	5	21.74	8	34.78	6	26.09	4	17.39

Table 6: Frequency and percent distribution of the cognitive strategies of organization used by the student teachers in solving mathematical problems

Meta-cognitive Strategies

There are two types of metacognitive strategies revealed in this study. These are the critical thinking and self- regulation.

Critical Thinking

The critical thinking among student teachers was shown through having estimated outcome, relating problems in daily life, selecting or choosing only important numbers or details in a problem and asking one's self if the answer makes sense.

In addition, Table 7 shows the frequency and percent distribution of metacognitive strategy of critical thinking used by the student teachers in solving mathematical problem-solving. The table reveals that almost everyone used critical thinking in solving mathematical problem-solving. Only one among the 23 respondents claimed that s/he compares the difference between the teacher's explanation and textbook content and draw a conclusion referring to the task.

Metacognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I usually question what I heard or what I learned in math class, and judge if this information is persuasive.	0	0	6	26.09	8	34.78	7	30.43	2	8.70
I make the math class materials as a start point and try to self-develop my own viewpoint to the topics.	0	0	4	17.39	12	52.17	6	26.09	1	4.35
I combine my own idea into the math class learning.	0	0	3	13.04	13	56.52	4	17.39	3	13.04
I try to find out another efficient way to solve the problem when I hear some ideas or some solutions.	0	0	3	13.04	6	26.09	12	52.17	2	8.70
I use a real example to verify the math theory conclusion.	0	0	6	26.09	10	43.48	6	26.09	1	4.35
I compare the difference between the teacher's explanation and textbook content.	1	4.35	4	17.39	11	47.83	5	21.74	2	8.70
I select relevant materials to solve the problem.	0	0	1	4.35	14	60.87	5	21.74	3	13.04
I make correct use of units	0	0	3	13.04	9	39.13	9	39.13	2	8.70
I make notes related to the problem	0	0	6	26.09	9	39.13	7	30.43	1	4.35
I monitor the on-going problem-solving process and change plan if necessary	0	0	4	17.39	10	43.48	9	39.13	0	0
I summarize the answer and reflect on the answer	0	0	6	26.09	9	39.13	7	30.43	1	4.35
I draw a conclusion referring to the task	1	4.35	5	21.74	10	43.48	4	17.39	3	13.04
I relate a future problems	0	0	5	21.74	8	21.74	10	43.48	0	0
I relate the given problem to other problems	0	0	0	0	7	30.43	14	60.87	2	8.70

Table 7: Frequency and percent distribution of the metacognitive strategies of critical thinking used by the student teachers in solving mathematical problems

Self-regulation

Student teachers reveal that they used self-regulation through answering the question, "how do you know that you have solved the problem correctly? What are your bases? And what makes you think it is correct?"

Student teachers associated getting the correct answer in checking their answers. If the answer matches with their checking, they are confident that the answer is correct. Some claim that they just know that it is correct because nothing is bothering them anymore or they are just confident that the answer is correct. Others just wait for the result if they are correct or not.

Table 8 shows the frequency and percent distribution of cognitive strategy of regulation used by the student teachers in solving mathematical problem-solving. The table reveals that almost all of student teachers responded sometimes true of me until always or almost true of me while few or almost nobody responded never or only rarely true in me. Thus, this shows that student teachers used their metacognitive strategies self-regulation in solving mathematical problems.

Metacognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I will go over to find out where the problem is.	0	0	3	13.04	12	52.17	8	34.78	0	0
I set up my own target and follow the agenda I make.	0	0	5	21.74	6	26.09	12	52.17	0	0
I reorganize and clarify the confused points after class.	1	4.35	5	21.74	6	26.09	7	30.43	4	17.39
I check my answer again after I finish the question.	0	0	2	8.70	9	39.13	7	30.43	5	21.74
I list related formula first.	0	0	3	13.04	10	43.48	8	21.74	2	8.70
When I make the wrong math answers, I will clarify whether this conceptual mistake or miscalculation.	0	0	6	26.09	7	30.43	6	26.09	4	17.39
I am correct in my calculations	0	0	6	26.09	11	47.83	6	26.09	0	0
I check my calculations by calculating again	0	0	7	30.43	6	26.09	7	30.43	3	13.04
I check the answer with the estimated outcome	0	0	3	13.04	13	56.52	7	30.43	0	0
I reflect on what went well and how the tasks were solved	0	0	7	30.43	7	30.43	6	26.09	3	13.04

Table 8: Frequency and percent distribution of the metacognitive strategies of self-regulation used by the student teachers in solving mathematical problems

Other Strategies

Other strategies were also revealed in this study such as prediction/orientation, planning, monitoring, and evaluating. These strategies were actually overlapping cognitive and metacognitive strategies discussed as classified by the action undertaken by student teachers as a part of the process of solving mathematical problems.

Prediction/Orientation

Prediction/orientation was revealed by the student teachers by analyzing the problem, again and again, underlining and selecting important details in the problem, drawing of the problem statement and having estimated outcomes which were categorized as rehearsal, elaboration, organization and critical thinking respectively. Prediction/orientation is shown through skipping difficult items and returning after solving the easy problems.

Moreover, Table 9 shows the frequency and percent distribution of prediction/orientation used by the student teachers in solving mathematical problem-solving. The table reveals that only one, two or nobody responded that they never used the other strategies presented to the student teachers. It also shows that only one or 4.35% responded to some selected items such as I underline important words in the word problem, I write down with my own words what I already knew, I have some idea or estimates of the possible outcomes, I select relevant steps to solve the problem, and two student teachers responded that they never underline important words in the word problem. Still, the majority responded that if not always, at least sometimes or even half of the time they used the other strategies presented in the table. Thus, this shows that student teachers use their other strategies of prediction/orientation.

Metacognitive Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I underline important words in the word problem	2	8.70	1	4.35	10	43.48	6	26.09	4	17.39
I select the relevant information needed to solve the problem	0	0	3	13.04	8	21.74	9	39.13	3	13.04
I read the task again to comprehend it better	0	0	3	13.04	5	21.74	7	30.43	8	21.74
I write down with my own words what I already knew	1	4.35	6	26.09	7	30.43	6	26.09	3	13.04
I put the information needed to solve the problem together	0	0	1	4.35	10	43.48	9	39.13	3	13.04
I write down with my own words what was asked for	1	4.35	5	21.74	9	39.13	4	17.39	4	17.39
I reflect on the works carefully and slowly on difficult exercises and fast on easy parts	0	0	3	13.04	7	30.43	12	52.17	1	4.35
I have some ideas or estimates of the possible outcome	1	4.35	3	13.04	10	43.48	9	39.13	0	0
I select relevant steps to solve the problem	1	4.35	5	21.74	7	30.43	9	39.13	1	4.35
I make a drawing related to the problem	0	0	2	8.70	7	30.43	10	43.48	4	17.39

Table 9: Frequency and percent distribution of prediction/ orientation used by the student teachers in solving mathematical problems

Planning

Student teachers actually planned before solving the given problem. This was shown through the act of underlining or selecting important details, calculating or estimating outcome and others. These actions were also classified as elaboration and critical thinking respectively.

In addition, Table 10 shows the frequency and percent distribution of planning used by the student teachers in solving mathematical problem-solving. The table reveals that all of the student teachers responded sometimes true of me until always or almost true of me while few or almost nobody responded never or only rarely true in me. Thus, this shows that student teachers used their cognitive strategies of planning in solving mathematical problems.

Other Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I select relevant numbers/data to solve the problem	0	0	2	8.70	6	26.09	10	43.48	5	21.74
I select the calculations that will be needed to solve the problem and to estimate the possible outcome	0	0	3	13.04	11	47.83	3	13.04	6	26.09
I select relevant materials to solve the problem.	0	0	1	4.35	14	60.87	5	21.74	3	13.04
I take time to design an action plan before actually calculating	0	0	4	17.39	6	26.09	9	39.13	4	17.39

Table 10: Frequency and percent distribution of planning used by the student teachers in solving mathematical problems

Monitoring

Through undergoing to the process, the student teachers strictly follow the whole step-by-step process. This is through solving repeatedly and remembering if they have encountered similar problems before. These were also classified as rehearsal. However, the student teachers also stated during the interview that they monitor their work to check progress, comprehension, and production.

In addition, Table 11 shows the frequency and percent distribution of monitoring used by the student teachers in solving mathematical problem-solving. The table reveals that only one responded never or rarely true of me in the item, I am correct in the calculation in using other strategies presented to the student teachers. It also shows that the student teachers sometimes used or always/almost used almost all of the other strategies showing monitoring presented to them. Thus, this shows that student teachers used monitoring in solving mathematical problems.

Evaluation

The student teachers who use metacognitive strategies such as organization, critical thinking, and elaboration also assess how well they accomplished their task of solving and how well they used learning strategies like making connections between parts of the problem; relating the problem in a sample in daily life and asking one's self if the answer makes sense; asking one's self some questions or talking to one's self; and checking answer respectively. These allow them to decide how effective the strategies were and to identify changes that they will make next time.

In addition, Table 12 shows the frequency and percent distribution of evaluation used by the student teachers in solving mathematical problem-solving. The table reveals that almost all used evaluation in solving mathematical problem solving except for the item on drawing a conclusion referring to the task.

Other Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I adhere to the plan systematically	0	0	2	8.70	10	43.48	10	43.48	1	4.35
I am correct in my calculations	1	4.35	5	21.74	11	47.83	6	26.09	0	0
I make correct use of units	0	0	3	13.04	10	43.48	9	39.13	1	4.35
I make notes related to the problem	0	0	6	26.09	9	39.13	7	30.43	1	4.35
I orderly take note of problem-solving steps	0	0	4	17.39	11	47.83	5	21.74	3	13.04
I do not forget problem-solving steps	0	0	7	30.43	12	52.17	4	17.39	0	0
I follow the sequences of problem-solving steps orderly	0	0	5	21.74	13	56.52	5	21.74	0	0
I act according to the plan	0	0	5	21.74	13	56.52	5	21.74	0	0
I monitor the on-going problem-solving process and change plan if necessary	0	0	4	17.39	10	43.48	9	39.13	0	0
I check my calculation calculating again	0	0	7	30.43	6	26.09	7	30.43	3	13.04
I check the answer with the estimated outcome	0	0	3	13.04	13	56.52	7	30.43	0	0
I reflect on the answer and only if all is checked giving a clear, exact and precise answer	0	0	4	17.39	11	47.83	8	21.74	0	0

Table 11: Frequency and percent distribution of monitoring used by the student teachers in solving mathematical problems

Other Strategies	1- never or only rarely true in me		2- sometimes true of me		3- true of me about half the time		4-frequently true of me		5- always or almost always true of me	
	f	%	f	%	f	%	f	%	f	%
I summarize the answer and reflect on the answer	0	0	6	26.09	9	39.13	7	30.43	1	4.35
I reflect on what went well and how the tasks were solved	0	0	7	30.43	7	30.43	6	26.09	3	13.04
I draw a conclusion referring to the task	1	4.35	5	21.74	10	43.48	4	17.39	3	13.04
I relate a future problems	0	0	5	21.74	8	21.74	10	43.48	0	0
I relate the given problem to other problems	0	0	0	0	7	30.43	14	60.87	2	8.70

Table 12: Frequency and percent distribution of evaluation used by the student teachers in solving mathematical problems

Discussion

The student teachers' response to the questionnaire reveals that they used cognitive, metacognitive, other strategies for solving problems in mathematics. This was similar to the metacognitive strategy knowledge used in constructing a framework of metacognitive strategy knowledge of Gurat and Medula (2016) supported by Liu and Lin (2010) in their Mathematics Learning Strategies Scale. These strategies were also similar to the metacognitive and cognitive strategies found by Akyol, Sungur, and Tekkaya (2010) in science class. Other strategies such as prediction/orientation, planning, monitoring, and evaluation was also similar to Brown's (1978) four types of skills. Three kinds of cognitive strategies revealed were rehearsal, elaboration, and organization. These strategies were the same as the three of the five learning strategies described by Simsek (2006) as cited by Simsek and Balaban (2010). The rehearsal refers to the strategies such as rereading the problem, solving problems repeatedly and recalling past lessons to better understand the problem before trying to solve it. Student teachers took time in analyzing repeatedly which depends on the difficulty of the problem. This is similar to the comprehension monitoring of Schurter (2002) where readers of a mathematical problem must be able to comprehend the problem. However, some student teachers do not repeat solving the problem whenever they were given limited time. When student teachers were given parallel problems they repeatedly solved the problem using the same formula/method or they recalled the past lesson and applied the same method for attacking the problem. Another cognitive strategy was elaboration. Elaboration was shown through underlining and selecting important details such as words and given in the problem and asking own self-questions related to solving. Alternatives were also used by student teachers such as listing or singling out the important details or what they cannot understand. Student teachers asked themselves to identify if the given is connected with what is asked about the problem. Some of them asked themselves in their mind and others talked to themselves regarding the steps, if their answer was right or wrong, how they understood the problem or how they analyzed the problem. Lastly, the organization was also shown by making connections between parts of the problem, making a drawing of the problem statement, and breaking down the problem into pieces, making simple charts/tables to better organize what is asked in the problem. Problem solvers relate parts of the problem in order to decide which of the values in the given were needed or not. If a solver failed to connect the given, he might fail to get the correct answer, especially that some problems were tricky that missing numbers are needed to be solved first before solving for what was asked in the problem. Through this, the problem solver may decide what strategy/formula/method/steps should fit the question. Moreover, making a drawing of the problem statement was also evident especially if the given problem requires illustration before one can solve it. Drawing or making representation was one of the problem strategies of Hoon, Kee and Singh (2013) in learning mathematics and the solution drawing strategy and use of graphs of functions of Novotná, et al. (2014). Furthermore, the study Krawec et al. (2012) used the same term, cognitive strategy for improving math problem solving of middle school students with learning disabilities. However, their cognitive strategy was an intervention that motivated students to use several problem-solving strategies.

Two types of metacognitive strategies were critical thinking and self- regulation. The critical thinking among student teachers was shown through having estimated outcome, relating problems in daily life, selecting or choosing only important numbers or details in a problem and asking one's self if the answer makes sense. Problem solvers may or may not have estimated outcome depending on the depth of understanding of the problem. Some problems may not require the solution because it can be solved by relating the problem in real life. This strategy was similar to the concept of Goldman and Booker (2009) who used everyday practices in mathematics. In terms of self – regulation, student teachers checked their own answer. Some know that their answer was correct and others just wait for the result.

Other strategies were also revealed in this study such as prediction/orientation, planning, monitoring, and evaluating. These strategies overlap with the cognitive and metacognitive strategies. Prediction/orientation refers to analyzing the problem, again and again, underlining and selecting important details in the problem, drawing of the problem statement and having estimated outcomes which were categorized as rehearsal, elaboration, organization and critical thinking respectively. Planning refers to the act of underlining or selecting important details, calculating or estimating outcome and others. These actions were also classified as elaboration and critical thinking respectively. Monitoring refers to the systematic process of solving while solving repeatedly and remembering if they have encountered similar problems before. These were classified as rehearsal and were similar to the strategy of analogy of Novotná, et al. (2014). Solvers also checked progress, comprehension, and production. Lastly, evaluation refers to the assessment of accomplishment and decision on the effectiveness of strategies used.

The findings revealed that student teachers are applying the variety of problem-solving strategies in mathematics. Despite the strategies used, the result of the students in the mathematics problem set test did not show favorable scores even if the students obtained a grade of passing rating (77 to 97) in their Problem-Solving subject except for one student who incurred an incomplete (INC) mark. The strategies used by student teachers and their grades in Problem Solving subject suggest that these strategies are a contributory factor on the passing grades of the student teachers. This corroborates the result of the studies of Akyol, Sunur and Tekkaya (2010); and Simsek and Balaban (2010) on the significant contribution of metacognitive and cognitive strategies to students' achievement. However, when strategies are related to the scores in the given mathematics problem set, it contradicts the result of the studies of Akyol, Sunur and Tekkaya (2010); and Simyek and Balaban (2010).

Conclusion

The problem-solving strategies among student teachers officially enrolled in the Problem-Solving subject are cognitive, metacognitive and other strategies. Cognitive strategies used in problem-solving are rehearsal, elaboration, and organization Metacognitive strategies involved in problem-solving are critical thinking and self-regulation and other strategies involved are planning, monitoring, and evaluation. These strategies can be taught by the student teachers for their future students. It may also help their future students succeed in solving math problems by student teachers' prior knowledge and skills in strategies. The identified strategies could also be considered in making problem sets for the students for the improvement of the students. Future researchers can work on identifying the strategies that lead to correct answers and incorrect answers could be conducted to better understand how strategies in solving affect the students in understanding and answering mathematics problems. Since the result of this study suggests a positive influence of the strategies on the academic performance of the students, a more in-depth study using linear regression or correlations may be conducted to validate the result. They may also consider other factors that might affect students in solving mathematics problems such as student's attitudes, basic arithmetic skills, and retention to find the possible reason of the low scores of the student teachers when given mathematics problem set in different areas of math.

Acknowledgment

The author thanks Dr. Cesar T. Medula, Jr. her thesis adviser; Dr. Dominga C. Valtoribio, Dean of School of Graduate Studies, and to the distinguished panel members, namely: Dr. Moises Alexander T. Asuncion, Mrs. Analyn A. Guevara and Mr. Jay-ar Castriciones for their time, patience guidance, direction, interest and encouragements in the completion of this research.

References

Akyol, G., Sungur, S. and Tekkaya, C. (2010) 'The contribution of cognitive and metacognitive strategy use to students' science achievement', *Educational Research and Evaluation*, Vol. 16, No. 1, pp. 1-21. https://doi.org/10.1080/13803611003672348

Brown, A.L. (1978) 'Knowing when, where, and how to remember: A problem of metacognition', In Glaser, R. (ed) *Advances in Instructional Psychology*, Vol. 1. Hillsdale, NJ: Lawrence Erlbaum Associates, [Online], Available: http://www.eric.ed.gov/PDFS/ED146562.pdf [7 Jun 2011].

Carson, J. (2007) 'A problem with problem-solving: teaching thinking without teaching knowledge', *The Mathematics Educator*, Vol. 17, No. 2, pp. 7–14.

Cockcroft, W. (1982) *Mathematics counts. Report of the committee of inquiry into the teaching of mathematics in schools*, London: Her Majesty's Stationery Office, (ED.), [Online], Available: http://www.educationengland.org.uk/documents/cockcroft/ [10 Jul 2011].

Dendane, A. (2009) '*Skills needed for mathematical problem solving*', [Online], Available: http://www.analyzemath.com/math_problems/paper_7.html [3 Apr 2011].

Dewey, J. (1933). '*How we think*', Boston: D.C. Heath.

Glaser, B.G., and Strauss, A.L. (1967) '*The discovery of grounded theory: Strategies for qualitative research*', Chicago: Aldine.

Goldman, S. and Booker, A. (2009) 'Making math a definition of the situation: Families as sites for mathematical practices', *Anthropology and Education Quarterly*, Vol. 40, No. 4, pp. 369-387. https://doi.org/10.1111/j.1548-1492.2009.01057.x

Gurat, M. and Medula, C. Jr. (2016) 'Metacognitive strategy knowledge use through mathematical problem solving amongst student teachers', *American Journal of Educational Research*, Vol. 4, No. 2, pp. 170-189. https://doi.org/10.12691/education-4-2-5

Halmos, P. (1980) 'The heart of mathematics', *American Mathematical Monthly*, Vol. 87, No. 7, pp. 519-524. https://doi.org/10.2307/2321415

Hoon, T., Kee K. and Singh, P. (2013) 'Learning mathematics using heuristics approach', *Procedia - Social and Behavioral Sciences*, Vol. 90, pp. 862-869. http://dx.doi.org/10.1016/j.sbspro.2013.07.162

Koichu, B., Berman, A. and Moore, M. (2004) 'Promotion of heuristic literacy in a regular mathematics classroom', *For the Learning of Mathematics*, Vol. 24, No. 1, pp. 33-39.

Krawec, J., Huang, J., Montague, M., Kressler, B. and de Alba, A. (2012) 'The effects of cognitive strategy instruction on knowledge of problem-solving processes of middle school students with learning disabilities', *Learning Disability Quarterly*, Vol. 36, No. 2, pp. 80-92. http://dx.doi.org/10.1177/0731948712463368

Krulik, S., and Rudnick, J.A. (1987) '*Problem solving: A handbook for teachers*', (2nd ed.). Boston: Allyn and Bacon.

Liu, E. and Lin C. (2010) 'The survey study of mathematics motivated strategies for learning questionnaire (MMSLQ) for grade 10-12 Taiwanese', *The Turkish Online Journal of Educational Technology*, Vol. 9, No. 2, pp. 221-223.

Novotná, J., Eisenmann, P., Přibyl, J., Ondrušová, J. and Břehovský, J. (2014) 'Problem solving in school mathematics based on heuristic strategies', *Journal on Efficiency and Responsibility in Education and* Science, Vol. 7, No. 1, pp. 1-6. https://doi.org/10.7160/eriesj.2013.070101

Polya, G. (1988) '*How to solve it: A new aspect of mathematical method*', (2nd ed.), Princeton, NJ: Princeton University Press.

Reiss, K. and Renkl, A. (2002) 'Learning to prove: The idea of heuristic examples', *Zentralblatt für Didaktik der Mathematik (ZDM),* Vol. 34, No. 1, pp. 29-35. https://doi.org/10.1007/BF02655690

Reys, E, Lindquist, Lambdin D.V., Smith, N.L. and Suydam, M.N. (2001) '*Helping children learn mathematics*', (6th ed.), John Wiley and Sons, Inc., New York.

Schoen, H.L. and Charles, R. I. (2003) '*Teaching mathematics through problem solving*', National Council of Teachers of Mathematics (NCTM) catalog.

Schoenfeld, A. (1992) '*Learning to think mathematically: Problem-solving, metacognition, and sense making in mathematics*', In D.A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 165–197). MacMillan, New York.

Schurter, W. (2002) 'Comprehension monitoring: an aid to mathematical problem solving', *Journal of Developmental Education*, Vol. 26, No. 2, pp. 22-33.

Simsek, A. and Balaban, J. (2010) 'Learning strategies of successful and unsuccessful university students', *Contemporary Educational Technology*, Vol. 1. No. 1, pp. 36-45.

Simsek, A. (2006) '*Bilissel stratejilerin ogretimi [Teaching cognitive strategies]*', In A. Simsek (Ed.), Icerik turlerine dayali ogretim (pp.181-208). Ankara: Nobel.

Stacey, K. (2005) 'The place of problem solving in contemporary mathematics curriculum documents', *The Journal of Mathematical Behavior*, Vol. 24, pp. 341. https://doi.org/10.1016/j.jmathb.2005.09.004

Stanic, G. and Kilpatrick, J. (1989) '*Historical Perspective on Problem Solving in the Mathematics Curriculum*', In R. Charles and E. Silver (Eds.), The teaching and assessing of mathematical problem solving (pp. 1-22), Reston, VA: National Council of Teachers of Mathematics.

Strauss, A., and Corbin, J. (1998) '*Basics of Qualitative Research: Grounded theory procedures and techniques*', (2nd ed.). London: Sage.

Wilson J., Fernandez M. and Hadaway, N. (2011) '*Mathematical problem solving*', [Online], Available: http://jwilson.coe.uga.edu/emt725/PSsyn/Pssyn.html [8 Aug 2011].

Appendix A

Mathematics Motivated Strategies Learning Questionnaires

Name:________________________________

This questionnaire has a number of questions about your metacognitive strategy knowledge.

There are many different strategies that good problem solvers use to solve a problem. It depends on the strategy which you may and may not be aware of.

Rate yourself by checking the box which you think is the most appropriate to you. Numbers below correspond to the following response.

1- never or only rarely true in me

2- sometimes true of me

3- true of me about half the time

4- frequently true of me

5- always or almost always true of me

Do not spend a long time on each item; your first reaction is probably the best one. Please answer each item. Do not worry about projecting a good image. Your answers are CONFIDENTIAL.

Be honest as you are in choosing the answer. This is not an evaluation.

No.	Statement	1	2	3	4	5
A.	**Before I begin, solving a problem,**					
1	I analyze the problem again and again.					
2	I mark-up the important lines for concepts organization.					
3	I underline important words in the word problem					
4	I select relevant numbers/data to solve the problem					
5	I adhere to the plan systematically					
6	I relate the given problem to other problems					
7	I take time to design an action plan before actually calculating					
8	I have some idea or estimates the possible outcome					
9	I compare the difference between the teacher's explanation and textbook content.					
10	I ask questions to myself to make sure that I understand the math materials content					
11	I repeatedly practice similar question types.					
12	I study the class notes and textbook again and again.					
13	I make the math class materials as a start point and try to self-develop my own viewpoint to the topics.					
14	I reorganize and clarify the confused points after class.					
15	I try searching for patterns or symmetry in order to find the correct answer like thinking of an easier problem than doing the given task.					
16	I read the task again to comprehend it better					
17	I select relevant materials to solve the problem.					
18	I make notes related to the problem					
19	I write down with own words what was asked for					
20	I select the relevant information needed to solve the problem					
21	I combine my own idea into the math class learning.					
22	I memorize the important and key math formula to remind me of the important part of my math class					
23	I link the class notes to textbook examples to improve my understanding.					
24	I read through the class notes and textbook and find out the most important parts.					
25	I read through the class notes and mark up the important parts.					
26	I categorize the easy-hard type questions of every exam.					
27	I try to find out another efficient way to solve the problem when I hear some idea or some solution.					
28	I set up my own target and follow the agenda I make.					
29	I list related formula first.					
30	I divide the problems into parts or I solve in general.					
31	I write down with own words what is already know					
32	I select relevant steps to solve the problem					
33	I orderly take note of problem-solving steps					
34	I relate a future problems					
35	I make a drawing related to the problem					
36	I put the information needed to solve the problem together					
B.	**While solving the problem,**					
37	I usually question what I heard or what I earn in math class, and judge if this information is persuasive.					
38	I know how and when to add, subtract, multiply and divide.					
39	I used trial and error when I don't know the formula of the problem.					
40	I have my own tactics in solving a problem					
41	In order to get the right answer, I have to follow the method step by step.					
42	I combine my own known knowledge with the learning materials.					
43	I make simple charts and tables to help me in organizing my math class materials.					
44	I am aware of what „borrowing" means in subtracting numbers.					
45	I select the calculations that will be needed to solve the problem and estimating a possible outcome					
46	I visualize the scenario in the problem by drawing, hoping to see what is really asked about the problem.					
47	I know how to manipulate the general formula to arrive at a certain formula on getting what is missing in the problem.					
48	I use arithmetic in solving the problem.					
49	I know what "carrying" means is in addition and how to use it.					
50	I use strategies which provide a definite and certain way to reach a goal.					
51	I try using different strategies like guess and check, diagrams and others in solving problems trying to bring out the answer even if I am not sure.					
52	I act according to the plan					
53	I am correct in my calculations					
54	I reflect on works carefully and slowly on difficult exercises and fast on easy parts					
55	I make correct use of units					
56	I do not forget problem-solving steps					
57	I follow the sequences of problem-solving steps orderly					
58	I monitor the on-going problem-solving process and change plan if necessary					
C.	**After I've arrived at the answer,**					
59	I do my best to link relative portions of math and other subjects.					
60	I go over to find out where the problem is.					
61	I summarize the answer and reflecting on the answer					
62	I reflect on the answer and only if all is checked giving a clear, exact and precise answer					
63	When I make the wrong math answers, I will clarify whether this is a conceptual mistake or miscalculation.					
64	I draw a conclusion referring to the task					
65	I reflect on what went well and how the tasks were solved					
66	I check my calculation calculating again					
67	I check my answer again after I finish the question.					
68	I use a real example to verify the math theory conclusion.					
69	I go over the formula and important concepts by myself.					
70	I find out any sample in daily life to link with math materials.					
71	I check the answer with the estimated outcome					
72	I repeatedly practice similar question types.					

Appendix B

Mathematical Problem Solving Set

Arithmetic

1. After the first 57 games of the UAAP season, the Blue Eagles have a winning percent of 0.561 and the Green Archers have a winning percent of 0.491. How many games behind the Blue Eagles are the Green Archers? (1pt)
2. If x is divided by 9, the remainder is 5. What is the remainder if 3x is divided by 9? (1 pt)

Algebra

3. The principal in a school decided that the number of scouts who could go camping would be greater than or equal to 100 but less than or equal to 140. She further wanted 2/7 to be from the fourth year scout and the rest would come in equal number from first, second and third year scouts.
   a. What minimum number of scouts from each year of the lower years could go? What is the maximum number?(2 pts)
   b. What is the minimum number of fourth year scouts that could go? What is the maximum number? (2 pts)
4. Gina and Bebs are practicing for a swimming competition. They are swimming back and forth to the swimming pool. Gina takes 2 minutes to swim the length of the pool while Bebs takes 3 minutes.
   a. If they begin together at the same end of the pool, after how many minutes will they start together from the same end? (1 pt)
   b. If they begin at the opposite end of the pool, after how many minutes will they start together from the same end? (1 pt)

Trigonometry

5. If points P, Q and R are the centers of the circles, and the circles have radii of 3,4,5 respectively, what is the perimeter of the triangle PQR? (1 pt)

Geometry

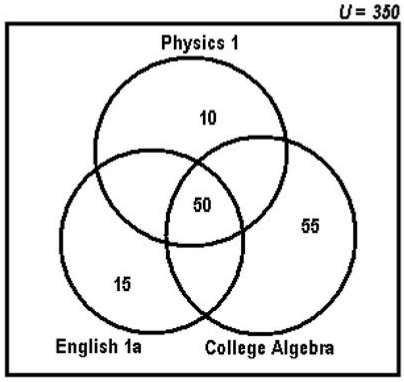
6. Most proofs are done by means of deduction: that is we proceed from the premises, step by step, to a conclusion. As we go from one step to the next step, we must have a reason for each step to show that it follows logically. The following is an example of the proof that does not obey the rules: even though the desertion appears to be correct, it is not. Can you find the error? (1 pt)

Statements		Reasons
1.	a = b	Given
2.	a^2=ab	Multiplying Both sides by a
3.	a^2-b^2 = ab – b^2	Subtracting b2 from both sides
4.	(a+b)(a-b) = b(a-b)	Factoring both sides
5.	$\frac{(a+b)(a-b) = b(a-b)}{a-b}$	Dividing both sides by a- b
6.	(a +b) = b	Result of Step 5
7.	b+b = b	Substituting b for a
8.	2b = b	Combines b + b
9.	2b/b = b/b	Dividing both sides by b
10.	2=1	Result for step 9

Q.E.D.

Sets

7. Consider the given information on the right regarding the number of enrolled students in three major subjects such as College Algebra, Physics, and English 1a. There are 350 students enrolled in these subjects. 65 of which are enrolled both in Physics and College Algebra, 70 of which are enrolled both in College Algebra and English 1a and 75 of which are enrolled both in Physics and English 1a.
   a. How many students are enrolled in College Algebra? (1 pt)
   b. What is the total number of students enrolled in English 1a? (1 pt)
   c. The number of students enrolled in Physics 1 is ___. (1 pt)
   d. The total number of students enrolled in Physics 1 and College Algebra is ____. (1 pt)
   e. How many students are enrolled in Physics 1a and College Algebra but not enrolled in both subjects? (1 pt)



Probability

8. In how many ways can 3 boys and 3 girls be seated in a row if:
   a. They may sit anywhere? (1 pt)
   b. The girls and boys must alternate? (1 pt)

Number Theory

9. What is the maximum number of positive consecutive integers that can be added together before the sum exceeds 5 000? (1 pt)

Puzzle Problem/ Logic

10. Three couples all like sport. Gill is a captain of the soccer team, Bill is a star basketball player and Neil is a good swimmer. However, Neil's wife cannot swim. Carolyn plays golf: Mylene, who by the way is Neil's sister, is a good dancer and Jennelyn, whose husband is very short, is an expert diver. Who is married to whom? (3 pts)

Appendix C

Semi-Structured Interview Protocol

Introduction:

Hi, I'm Ms. Melanie G. Gurat. I am so glad you have decided to participate in this study. The purpose of this project is to better understand your thinking in Mathematical Problem Solving.

And I want you to feel free in using the dialect in answering each question. The answers that you will give in this interview will help a lot in my research so please do not hesitate to answer them as honestly as you can. If I stop you from asking a question, I am not actually disagreeing but only trying to gain a better understanding of the way you think about some things. I'll be recording and videotaping this interview and transcribing it, but the information you will share with me will be strictly confidential. The answers you will give in this interview will not affect your class evaluation. If there are questions that are not clear to you, feel free to ask me. Do you have any questions? (There will be pre-interview questions to be asked to establish rapport with the students and let them feel comfortable with the researcher.)

Interview Questions:

1. What do you know about mathematical problem-solving?
2. How do you solve mathematical problems?

   Probing Question if in case the answer of the respondent is more technical: What are the processes you usually use in solving math problems?
   a. If you are familiar with the problem? (you know the formula)
   b. The problem is new to you or you are not familiar?
3. What is the first thing you do?

   a. Do you analyze the problem again and again?

   b. Do you make connections between parts of the problem? When? Why?

   c. Do you underline and select important details such as words and given numbers? Do you usually use all the information in the problem to solve what is unknown? When? Why?
4. How do you know you have understood the problem?
   a. Do you master the problem by solving the problem repeatedly?
   b. Do you make a drawing of the problem statement?
   c. Do you have an estimated outcome?
   d. Do you relate the problem in the sample in daily life?
5. Do you also try using different strategies in solving varied mathematical investigation problems?? What are those strategies? Why do you prefer to use them?
6. How do you select a strategy in solving a specific problem?
   a. Do you try to remember whether you had worked on the problem similar to this before?
   b. Do you ask yourself other questions to understand the problem? What are those questions? Why do you ask such questions? Do you usually ask questions or talk to yourself throughout the problem-solving process?
   c. Do you break down the problem into pieces, make simple charts/tables to better organize what is asked in the problem?
7. Do you usually use all the information in the problem to solve what is unknown? Why?
8. Once you have arrived at an answer, what do you usually do? How often? Why?
   a. Do you check your answer again?
      i. Do you look back
      ii. Do you substitute your answer with the formula/ recheck the algorithmic computations
      iii. Verify it using other strategies?
   b. Do you ask yourself if your answer makes sense?
9. How do you know you have solved the problem correctly? What are your bases? What makes you think it is already correct?
10. Any concluding statements regarding your experience in solving mathematical investigation problems
    a. During solving the problem, and you encountered difficulty (describe the character of difficulty)
    b. During solving the problem, you found a mistake and corrected it (describe the mistake)

Note: Probing questions will depend on students' responses on each question above.

STUDENTS WHO HAVE UNSUCCESSFULLY STUDIED IN THE PAST – ANALYSIS OF CAUSES

Petr Mazouch[1]✉, Veronika Ptáčková[2], Jakub Fischer[2], Vladimír Hulík[3]

✉[1]University of Economics, Prague, Czech Republic, mazouchp@vse.cz

[2]University of Economics, Prague, Czech Republic

[3]Ministry of Education, Youth and Sport, Prague, Czech Republic

Highlights

- *Students who definitely do not work at their preferred college are 6.3 times more likely to have unsuccessfully completed tertiary education in the past than students who definitely study in their preferred university*
- *Men are 1.426 times more likely to have an unsuccessful past tertiary education than women*
- *Students with a health disadvantage are 1.3 times more likely to have an unsuccessful past tertiary education than students who do not suffer from health complications*

Abstract

With the increase in the number of university students, the number of those who do not finish successfully the tertiary education is also increasing. The article uses a specific data source and analyses only a part of the group of unsuccessful students who re-enroll. This is a specific group of students - they did not finish the tertiary study in the past, but after some time they returned to education. The aim of the paper is to find significant factors that influence the decision whether the student changes the studied school or field of study. Factors will be searched using decision trees and binary logistic regression. Both methods were significant for gender and the fact that a student is studying his preferred university. Logistic regression adds to the student's health disadvantage. The data were obtained from the EUROSTUDENT survey, which was held in the Czech Republic in 2016 under the auspices of the Ministry of Education, Youth and Sports. The results can be used to identify a risky candidate or student at the beginning of tertiary education.

Article type

Full research paper

Article history

Received: March 15, 2018

Received in revised form: August 10, 2018

Accepted: September 21, 2018

Available on-line: September 30, 2018

Keywords

Drop-out, EUROSTUDENT, failed studies, human capital, tertiary education, universities in the Czech Republic, unsuccessful students

Mazouch P., Ptáčková V., Fischer J., Hulík V. (2018) "Students Who Have Unsuccessfully Studied in The Past - Analysis of Causes", *Journal on Efficiency and Responsibility in Education and Science, Vol. 11, No. 3*, pp. 65-71, online ISSN 1803-1617, printed ISSN 2336-2375, doi: 10.7160/eriesj.2018.110303.

Introduction

The phenomenon of recent years is the growth of the university-educated population in the Czech Republic. Nevertheless, together with the interest in tertiary education, there is also a growing number of those who fail to complete university studies. This topic is not only about the Czech Republic but also about other EU countries.

In this article, we focus on a specific group of students who have re-enrolled their studies again when the previous studies were unsuccessful. The data from the international survey EUROSTUDENT VI organized by the Ministry of Education, Youth and Sports were used for the analysis. In order to find statistically significant factors of unsuccessful study in the past, we use following statistical methods: binary logistic regression and decision trees - specifically, the CART (Classification And Regression Trees) method. The results of both methods are compared and confronted with conclusions from foreign and Czech studies. This identifies factors which can help characterising a risky candidate or a student at the beginning of a course. The results may help to reduce the proportion of unsuccessful students, which could be interesting for a policy of tertiary education, as well as for study advisers of individual universities and faculties.

Literature Review

The general term for unsuccessful study is usually "drop-out". It does not distinguish whether it captures a course, a degree program or an educational level. There is no uniform definition of this term in the Czech Republic or the world. Most often, the drop-out is translated into Czech as "early departure from education" or "unsuccessful termination of education".

The drop-out calculation is often complicated. Problems can occur both due to the lack of a clear definition of the concept and the structure of the analysed data. International organizations do not analyse individual programs but levels of education. For example, the Organization for Economic Co-operation and Development (OECD) includes all students who have completed a given level of education without qualifying (Hraba, Hulík, Hulíková Tesárková, 2016). For Eurydice, which deals with the situation of higher education institutions across countries, the following definitions have been used for the Czech Republic: "Unsuccessful termination of tertiary education means a situation when the student fails to appear again as a tertiary student after another unsuccessful graduation for the next three years" (Hraba, Hulík, Hulíková Tesárková, 2016).

The Czech Ministry of Education, Youth and Sport recommends calculating the cohort rate of failure. This rate is associated with the registration year of study. We can calculate it as the ratio between the number of unsuccessfully completed studies in each year of study and the total number of studies commenced in that year of enrolment. The problem is that is focused on the study, not on the student. (Ministry of Education, Youth and Sports, 2017)

At the national level in the Czech Republic, we can use the cohort rate of failure which is connected with the student. It is monitored all years in the tertiary education. This rate is calculated as the ratio between the number of unsuccessfully completed studies in the concrete cohort (and concrete level of study) and the total number of students who come to the concrete level of study for the first time in the concrete year (Vlk et al., 2017). For more details about drop-out definition, see Vlk et al. (2017).

Some explanations of reasons for unsuccessful studies at universities are based on the theoretical model of student residence in an academic environment designed by Tinto (1975). Tinto's sociological-anthropological model states that a student successfully completes university studies, not only when properly fulfilling study duties but also actively integrating into natural social structures in the academic environment. Tinto points out that if the student more communicates with classmates and faculty, his/her chances of successfully completing the studies are increasing. It emphasizes the responsibility of the school to support the student's academic and social integration (Tinto, 1997).

Tinto (1997) identifies two types of study leaving. The first is the termination of studies because of insufficient learning outcomes (involuntary leaving) and the second voluntary leaving from studies, which can be affected by a number of factors. Tinto (1997) points out that the school should define its duties and obligations towards the student (as well as the student to school). In addition, he identified six basic conditions that support the success of the study: the duty of the school to enhance student success, student expectations, student support, feedback on student performance, student-to-student relationships, and student learning. Tinto (1999) says that the critical period of study is the first year.

Jensen (2011) divides factors into three levels: individual (academic performance, student attitude and satisfaction with study), institutional (conditions created by the school: pro-social climate in school, support services, awareness of student needs, opportunity to participate out-of-school activities) and external social standards (social support: support from parents, friends, schoolmates).

The German Center for Higher Education and Scientific Research[1] (Heublein, 2014) has drawn up a model that highlights the fact that unsuccessful completion of studies cannot be described as an individual failure or problem of the education system but as a complex problem that can be divided into three phases. The preliminary stage is affected by the social status and family background, the content of the study program, the study itself and socialization in the educational process. The second phase reflects the relationship between the internal (motivation, performance, psychological and physical possibilities of the student) and external (study, accommodation) factors. The final decision is the third phase. (Heublein, 2014).

When analysing the effects of terminating studies at German public schools, the factors were divided into three groups: predisposing (social and demographic factors, personality traits, the initial level of knowledge and motivation), important life events (work and family responsibilities), and institutional factors (methods of studies, teachers, administrative support). Fully-employed students, migrants, and women - who have higher expectations than men in the study program and the environment - are included in the risk group. On the contrary, older students with higher motivation for professional and personal growth, and students with a child have a higher odd to graduate (Stoessel et al., 2015).

Wolter, Diem and Messer (2014) found a higher drop-out rate for men and older students. It also depends on the education and employment of the student's parents, the results of admissions, integration, and motivation. The study highlights the influence of the Bologna process when a lot of master's programme was divided into bachelor and follow-up master's programme. Due to that change, the rate of unsuccessful women decreased.

Kingston (2008) emphasizes emotional intelligence and satisfaction with the learning environment. Vnoučková et al. (2017) point out the importance to have student's feedback not only about ongoing subjects but also at the end of the subject, but also about all their tertiary studies – it can help to increase the quality of the university and student's satisfaction. Kearney and Levine (2016) address the problem of income differentiation and early school leaving. They point out that boys are more responsive to family and economic disadvantages. They suggest that governments should invest more in human capital (low-income students) at secondary schools. Early intervention can thus positively influence decisions on further study.

Pikálková, Vojtěch and Kleňha (2014) confirmed that the number of unsuccessful students has risen in recent years. They assume that half of the students - who attended college in 2012 - have had not finished it. Higher risk of abandonment is attributed to secondary school postgraduate graduates and secondary vocational schools with graduation. According to the authors, the rate of departures varies with the field of study. Students of technical disciplines are more likely not to have over-pressure in admissions. Mathematics, physics or agriculture students also leave more often.

Fučík and Slepičková (2014) emphasize that students who went to study as a so-called deferred choice are more likely to leave (the students went to college for which they were admitted and then left). Again, this is a conflict between expectation and reality. Also, family and professional opportunities have influence. Charvát et al. (2014) stress the importance of interest and satisfaction with the study. Rubešová (2009) shows the connection between the success of the university studies with the result of the admission procedure and the secondary school achievement. Konečný, Basl and Myslivečel (2010) confirmed these results. They say that students from grammar schools are less risky because they have better preparation for entrance examinations and study.

Hloušková (2014) points to internal factors of incomplete university studies, low socio-economic and cultural status, unfavourable family environment, fostering and educational aspirations of parents. External factors are the difficulty of study, university environment, teaching teacher skills and the rules of the educational institution. In addition, she mentions the influential events of pregnancy, injury, illness or poor school choice.

Menclová, Pacnerová and Vacek (2008) came up with the term "amotivation", which indicates little or no motivation to study at students who do not know what jobs they want to do in the future. They begin to study the field for which they successfully passed entrance examinations. They also work with the concepts of "leaving behind something" and "leaving as an escape". "Leaving for something" captures a situation when a student stops studying for work or family reasons. "Leaving as an escape" capture the termination of studies that arose from stress, crisis situations, conflict, inability to combine the field of study with personal interests, abilities, and talents.

Data – EUROSTUDENT VI

The EUROSTUDENT - international project - seeks to obtain comparable data on the social dimension of European higher education. The survey should clarify issues related to the living conditions and attitudes of students in bachelor and master

1 Deutsches Zentrum für Hochschul-und Wissenschaftsforschung

programs taught in Czech in five key areas (German Centre for Higher Education Research and Science Studies, 2017):

- the permeability of studies,
- student relationship to school,
- living conditions of students,
- the foreign mobility of students and language skills,
- students with disabilities.

For the first time, EUROSTUDENT was organized by the European Higher Education Area (EHEA) in 1994. In recent years, the country has been striving to maximize EHEA, provide high-quality higher education, increase graduate employment, and improve student international mobility as a tool for improving learning outcomes. The financial and economic crisis has affected student living conditions (Hauschild et al., 2015). This is one of the reasons why today ministers seek public funding for higher education, reduce inequalities, and provide quality support to students during their studies, individual consultations and the diversity of the studied subject areas. They want to increase employment and student international mobility (Hauschild et al., 2015).

The sixth wave of this international survey was held in 2016. Respondents were: public, state and private universities in the Czech Republic which have accredited bachelor, master or postgraduate courses taught in the Czech language. Over 230,000 students were approached within the project. 22,207 students entered the questionnaire, but 16,602 students completed it. After a detailed analysis of the data, fifty-one questionnaires that were not filled completely but fulfilled minimum requirements were added to the calculation. Weights were assigned on the basis of data from the United Students Register Information System containing gender, age, type of study program, and college (Fischer et al. 2016).

The variables from EUROSTUDENT VI, which were selected on the basis of a literature review, came into the analysis. There were the social and demographic factors with which a student came to university and which could have influenced unsuccessful studies in the past:

- type of high school,
- gender,
- the social status of parents,
- mother's highest education,
- father's highest education,
- mother's job,
- father's job,
- the answer to the question: "Was your university preferred option?",
- health handicap.

A variable *unsuccessful study in the past* is a dependent variable that can acquire two values: "yes" and "no". Unsuccessful college studies are defined in the EUROSTUDENT VI survey as termination of study without a title (failure to meet study requirements, termination at their own request, etc.).

Methods

Two statistical methods were used to find significant factors: logistic regression and decision trees. The methods were chosen for the binary explanation of the variable and the character of the task solution. According to available sources, EUROSTUDENT data were processed for the first time in this way.

Decision trees

Structure of decision trees looks like a reversed tree that displays a hierarchical set of relationships between dependent and independent variables. The method can be used not only to classify individuals but also to classify a set where the starting population (e.g. respondents) is divided into smaller homogeneous groups (respondents who are characterised by some property). In addition, this method detects dependence between dependent and independent variables (Vild, 2012). Trees are formed by using different algorithms – they are different in optimal cleavage. In this case, the Classification and Regression Tree (CART) method was used, which is good for categorical and regression tasks. Trees arise from a recursive binary division. At the beginning of tree formation, all observations are brought to one node (root). The observations are divided gradually into two daughter nodes based on the value and the predictor *X*. The division to the other nodes is binary again (Breiman et al., 1984). Predictor *X* should divide the dependent variable so that the values of the dependent variable inside the node resemble as much as possible but different as much as possible between the nodes. The homogeneity of a node is determined by the Gini index, entropy, or classification error (Komprdová, 2012).

Classification forest will be created by a combination of classification trees. The value of the predictor vectors is determined by each tree in the given class. Voting is determined by the classification function. Regressive forests that contain regression trees are generated by a similar procedure, the resulting regression function is calculated as the average of regression functions of individual trees (Klaschka, Kotrč, 2004).

Logistic regression

Logistic regression is used to find the best - meaningful model. This model describes the relationship between the dependent variable and the group of independent variables. Binary logistic regression is used in this analysis because the dependent variable has only two values. An easy interpretation of the results is an advantage of this method (Řeháková, 2000). In addition, the output can be described as a mathematical model. A model displays the relationship of the dependent variable to the other independent variables. A model allows for the stepwise selection of the independent variables (Tufféry, 2011, Hosmer, 2000).

Model quality

Model quality is evaluated as a whole (not as a component). The ability to predict effectively the values of the dependent variable using independent variables based on observed data means a quality. Among the methods which that model evaluates belong: classification table, ROC[2] curve, statistics (Cox-Snell determinant, Nagelkerk determination factor, 2LL) (Hosmer, Lemeshow, 2000). The classification table records the number of correctly and incorrectly classified objects. On the main diagonal, we can find correct classified objects. As a consequence of the classification table, we can calculate sensitivity and specificity in our logistic regression model. Sensitivity is the probability that the object with the positive answer is classified correct. Specificity is the probability that unsuccessful object is classified as unsuccessful. The graph, which illustrates the relation between sensitivity and specificity, is called ROC curve. *X*-axis values are calculated as (1 – *specificity*), *Y*-axis values are *sensitivity* (Betinec, 2006). The theoretical ROC curve for a random predictor (i.e. for a zero-discriminatory test) leads from the lower left to the top right corner. ROC curve is drawn in a unit square. The closer ROC curve is to the top left corner

2 Receiver Operator Characteristic

of the unit square, the better is the discriminative quality of the test (Tufféry, 2011).

Random forest is another option for verifying the quality of a model. It consists of one thousand trees with the same dependent variable. The difference is that each time the data are randomly divided into the training and test set. The software used assesses the importance of the variables involved, whichever is closest to the root node. Subsequently, according to significance, for the explained variable (unsuccessful study), it is sorted downwards according to predictive and confidential significance.

There are two metrics calculated during calculation: Mean Decrease Accuracy and Mean Decrease Gini. Mean Decrease Accuracy says how the accuracy decreases on average when the given tree model variable in the given forest is dropped. Mean Decrease Gini is related to the Gini index for that independent variable. The figure says how much variability, resp. diversity, of the dependent variable can the independent variable explain. A variable with a higher value brings better results.

The calculations were performed using the statistical program R.

Results

Main results from the survey Eurostudent VI

In the Czech Republic, one-fourth of college students have experience with unsuccessful studies (24.8%). These students could identify a combination of factors in the questionnaire which played a role in deciding to leave tertiary education. The most frequent reasons were: dissatisfaction with the content of the study (45.3%), high study intensity (38.6%), dissatisfaction with the quality of teaching (19.6%), lack of social integration (17.2%) and the fact that completed study was only a "backup option" (15.9%). Men left the university because they have a job opportunity or lack of social integration. Women left for the health and family reasons, and because their study was a backup option for them (Fischer et al., 2016).

Decision trees

We used a fixed set of the statistical program R, which states that the trees cannot be more complex than the edge-end metric. The tree was formed by randomly dividing the data into a training and test set. The training set contains seventy percent of the analysed data. The decision tree was created based on this set. The data from the test section was subsequently used to rank in the correct class dependent variable *unsuccessful study.*

The biggest influence on the experience with failed studies in the past had the answers: "*rather not*," "*certainly not*" to the question: *Was university (which you study nowadays) your preferred option?*[3]. Subsequently, the tree was divided by *gender*. More often, men leave and return to tertiary studies compared to women.

The quality of the model was evaluated by the classification table (Table 1) and the ROC curve. The decision tree (created by CART) has very good prediction capabilities.

		Predicted values	
		Yes	No
Real values	Yes	152	875
	No	119	3,828

Table 1: Classification table, Eurostudent VI (source: own calculation)

Out of 1,027 unsuccessful students, 152 were classified correctly and 875 were misclassified. It is 14.8%. Out of 3,947 successful students, 3,828 were classified correctly and 119 were misclassified. The prediction ability is 97.0%. The accuracy is the most important result. If we sum both the correct and incorrect classifications, we get 152+3,828=3,980 correct classified cases. The total sum of the objects is 4,974. We can calculate the accuracy as 3,980/4,974=0.8001. When we transform it into the percentages, the accuracy is 80.01%.

During calculation the metric Mean Decrease Accuracy (Figure 1), the biggest values were at independent variables: *Was university (which you study) your preferred option*, *father's highest education* and *mother's highest education*. When we remove the variable *Was the university (which you study) your preferred option* from the model, we can classify the wrong 151 students on the average. In the case we remove *father's highest education*, resp. *mother's highest education*, the misclassification can be 46, resp. 43, students on the average.

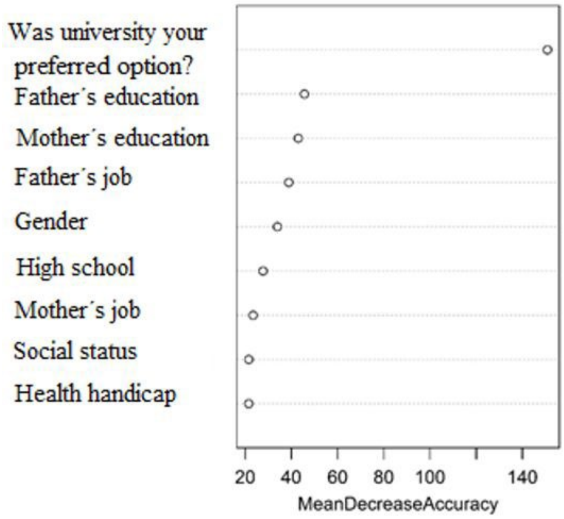


Figure 1: Mean Decrease Accuracy in a random forest, Eurostudent 2016 (source: own calculation)

The most important variables, according to the metric Mean Decrease Gini (Figure 2), were: *Was university (which you study) your preferred option* (282.97), *type of high school* (147.03) and *father's highest education* (143.03). We observe that the satisfaction with the university is a key classifier for drop-out. The result was to be expected because many studies reported the fact that it is important for a student to be in a college he wished to study and was not just a backup option.

The type of secondary school studied was the second major factor. The studies have confirmed that students who come from grammar schools or continue to study in a field of study (which they have at a specialized high school) have a better chance of completing tertiary studies successfully. The following three variables (*father's education, social status, mother's education)* can be summarized into one - the student's social background. Parents with tertiary education lead the child to study at the university. For their child, this is a logical step for getting a job. In addition, parents with higher education have usually better financial background than parents with basic education. For poorer students, the financial situation can be the reason why they prefer to go to work than to the university. The university and government should discuss more intensively about the financial support of these students.

Predictive ability of the forest should be higher than the prediction ability of the decision tree. For this reason, the rate has been established. The prediction ability was approximately the same as for the decision tree: 79.92%.

Model quality can be verified graphically using the ROC curve (Figure 3). Due to a large number of random trees in the random

3 The respondent can choose answers: „certainly yes", „rather yes", „rather not" and „certainly not".

forest, the sensitivity and specificity for the most accurate tree will be determined (red point in the Fig. 3 – based on Euclidean distance), which is closest to the upper corner of the ROC curve. **This sensitivity is 0.684 and the specificity is equal to 0.719**.

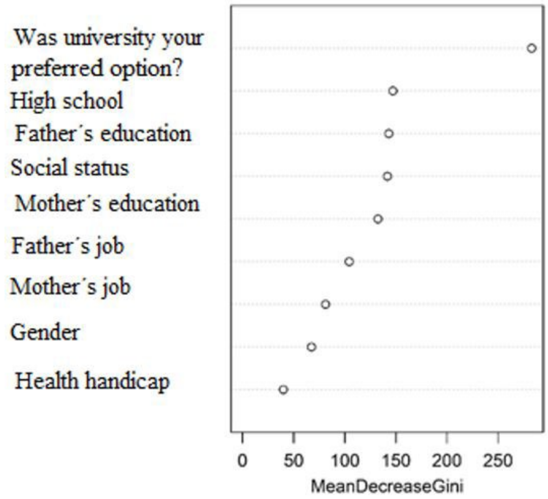


Figure 2: Mean Decrease Gini in a random forest, Eurostudent 2016 (source: own calculation)

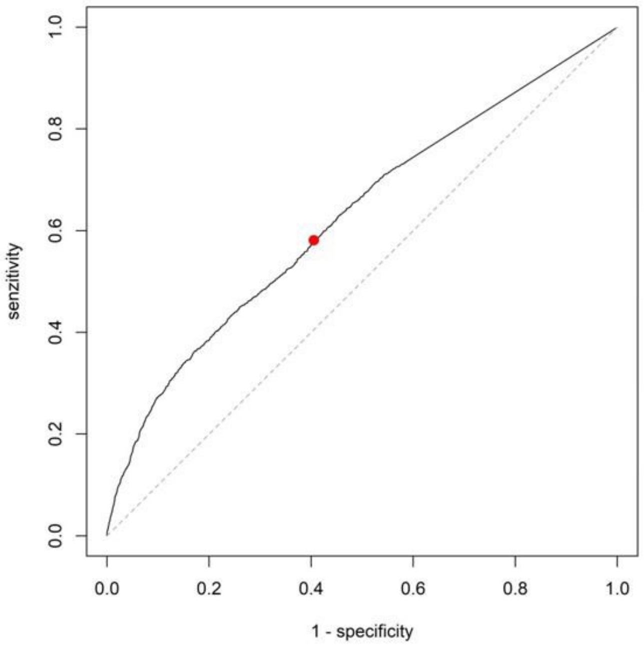


Figure 3: ROC curve in a random forest, Eurostudent 2016 (source: own calculation)

Binary logistic regression

Binary logistic regression was another way to find significant factors. Independent variables have been referenced to the reference category, which for each independent variable was the first category. The hypothesis was tested that there is no move between categories. In case of confirmation, the dependent variable in the model would be meaningless and could be removed. Conversely, the alternative hypothesis confirms its impact.

At the 1% level of significance, significant variables were identified: *gender, health disadvantage*, *education* and *employment of the mother*, *type of high school* and the answer to the question: "*Was university (which you study) your preferred option?"*.

Table 2 describes the results from binary logistic regression. In the first column (OR), we can see the ratio of probability which says the chance that student (with some concrete characteristic) failed in the past in comparison to the reference category of the question. The answer to the question: "*Was university (which you study) your preferred option?"* had the biggest impact on the experience of an unsuccessful study. Students who definitely do not study their preferred college (their answer is "certainly not") have a 6.3 times higher chance of not completing tertiary education in the past than students who certainly study in preferred university (their answer is "certainly yes"). Students who do not attend the preferred college (their answer is "certainly not") do not complete the study successfully in the past 3 times more often than students who have placed their college at the same time in the first place (their answer is "certainly yes").

The variable *Gender* has also the influence. The man has 1.4 times bigger chance that he fails during the studies than women. Disabled students have 1.3 times higher chance to have unsuccessfully completed university studies in the past than student without health complication.

	OR	*p*-value
Mother's education = PhD (reference category = max. elementary school)	1.576	0.000
Gender = Man (reference category = woman)	1.426	0.000
High school = postgraduate graduates secondary vocational schools without graduation (reference category = Secondary vocational secondary school – excluding lyceum)	0.454	0.000
High school = Multi-year gymnasium (reference category = Secondary vocational secondary school – excluding lyceum)	0.830	0.001
Health handicap = Yes (reference category = No)	1.254	0.000
Was university (which you study) your preferred option? = Rather yes (reference category = certainly yes)	1.436	0.000
Was university (which you study) your preferred option? = Rather not (reference category = certainly yes)	3.286	0.000
Was university (which you study) your preferred option? = Certainly not (reference category = certainly yes)	6.307	0.000

Table 2: **Binary logistic regression, Eurostudent VI (source: own calculation)**

Discussion

Existing data sources as the database of Ministry of Education, Youth and Sport allow us to analyse the relationship between surveyed variables. Our results show more than we can find in results of both statistical methods - decision trees CART and binary logistic regression - a subjective response to the question: "*Was the university you are studying your preferred option?*" was a significant variable. Students who definitely do not study (their answer is "certainly not") at their preferred college are 6.3 times more likely to have unsuccessfully completed tertiary education in the past than students who study definitely in their preferred university (their answer is "certainly yes"). Those who are not currently studying their preferred institution (their answer is "rather not") are 3.3 times more likely to have experience with unsuccessful study than those who are definitely studying at their preferred college (their answer is "certainly yes"). It is clear, therefore, that students, after failing to complete their studies, choose the "backup" option and prefer to study afterwards, which they do not indicate as preferred. On the other hand, this result indicates that students after their unsuccessful studying can find another university (or study program) but with much less motivation to study it because this is not his or her preferred choice.

Decision trees, as well as logistic regression, have confirmed that men have a higher degree of failure than women. Men are 1.426 times more likely to have an unsuccessful past tertiary education than women. Wolter, Diem and Messer

(2014) published the same conclusion and as we combine this information with result from the same Wolter's work that men more often study mathematical and technical disciplines and that Pikálkova, Vojtěch and Kleňha (2014) published that in the same study programmes not to have over-pressure in admissions it could be reason why some students (especially males) would underestimate the difficulty of the university studies.

Also, health disadvantage can play a role in whether a student has unsuccessfully completed tertiary education in the past. Those who are at a disadvantage are 1.3 times more likely than students who do not suffer from health complications. It is, therefore, less possible for health-disadvantaged students to hide their strength against other students, but it can also indicate that schools cannot work with the disadvantaged in such a way as to provide them with the necessary conditions, and these students then go to study elsewhere.

It seems as there is the wider definition of the second factor defined by Jensen (2011) – not only university as the institution should form student but also secondary school has to prepare the student for next studies and it should be moderated at a high school in line with this fact. A definitely supportive solution is to raise the awareness of graduates about the conditions of study at universities, compulsory subjects and graduate profiles, which could also help to increase the intensification of the relationship between the students of the high and secondary schools themselves. The greatest degree of learning failure is concentrated in the first year of study. This is referred to as a "deferred choice" - students are poorly informed and when they start studying, they decide whether to stay or not (The Ministry of Education, Youth, and Sports, 2014). One of the reasons why students do not attend their preferred school is that they could study during their previous studies, but for some reason - financial, family, they did not have the study responsibilities - they left school. Questionnaire EUROSTUDENT VI does not answer this question. Secondary schools should better shape the student in his / her expectations due to his / her abilities.

Conclusion

The article has set the objective to analyse the defined segment of unsuccessful students who got into the studies again. The use of the EUROSTUDENT VI data source allowed a deeper but significantly more limited analysis of the reasons and factors of leaving the study in general, which is comprehensively published in the Czech Republic by Vlk et al. (2017).

Policymakers should be able to answer the question whether the fact that students tend more often to study an less solicited field after the unsuccessful study is ok, especially if this likelihood is higher for a group of people with health disabilities.

In addition to existing studies, these analysed data also show that the current system is not optimized and leads to a number of disbalances. It is not a realistic goal for all students to study a preferred field, although, as a theoretical goal this may. That fact should be much more integrated into the decision-making process at high school than at present. Current study programs should be better described with the correct keywords. Candidates should have better information about the study, the study requirements and the subsequent application. It is difficult to select a study program by name. Usually, the program (its name) can be found at more universities but they have different content each time. Still, it may be appropriate to ask why students study non-preferred disciplines, ask and then seek for the answers to how to improve this situation.

Acknowledgement

This article was supported by the University of Economics, Prague under the name „The impact of social status on the attitude and behaviour of students at higher education institution." Grant, number [IGS/29/2017] together with the Institutional Support for Long-Term and Conceptual Development of Research and Science at the Faculty of Informatics and Statistics, University of Economics, Prague. The part of this article was published in the proceedings of conference Reproduction of Human Capital (Ptáčková, Mazouch and Fischer, 2017). The paper was called: "Is our society wasting human capital?".

References

Betinec, M. (2006) *Použití ROC křivek pro hodnocení klasifikátorů,* Prague: JČMF.

Breiman, L., Friedman, J.H., Olshen, R.A. and Stone, C.J. (1984) *Classification and Regression Trees,* New York: Wadsworth Inc.

Fischer, J., Vltavská, K., Brázdilová, M., Lipovská, H., Mazouch, P., Ptáčková V. and Šimková, M. (2016) *EUROSTUDENT VI Základní výsledky šetření postojů a životních podmínek studentů vysokých škol v České republice*, [online], Available at: http://www.msmt.cz/uploads/odbor_30/TF/Analyticke_materialy/Eurostudent/E_VI_zaverecna_zprava.pdf [28 Feb 2018].

Fučík, P. and Slepičková, L. (2014) ‚Studenti, kteří odcházejí: Kvantitativní analýza nedokončených vysokoškolských studií', *AULA*, Vol. 22, No. 1, pp. 24–54.

German Centre for Higher Education Research and Science Studies (2018) *EUROSTUDENT social dymension of European higher education: eurostudent*, [Online], Available: http://www.eurostudent.eu/index_html [14 Feb 2017].

Hauschild, K., Gwosć,C., Netz, N. and Mishra, S. (2015) *Social and Economic Conditions of Student Life in Europe*. Bielefeld: W. Bertelsmann Verlag GmbH & Co. KG.

Heublein, U. (2014) ‚Student Drop-out from German Higher Education Institutions', *European Journal of Education*, Vol. 49, No. 2, pp. 497–513. http://dx.doi.org/10.1111/ejed.12097

Hloušková, L. (2014) ‚Mám základní vzdělání. Příčiny a důsledky předčasných odchodů ze studia a ze vzdělávání', *Studia paedagogica*, Vol. 19, No. 2, pp. 11-38. http://dx.doi.org/10.5817/SP2014-2-2

Hosmer, D. W. and Lemeshow, S. (2000) *Applied logistic regression, 2nd edition*, New York: John Wiley & Sons.

Hraba, J., Hulík, V. and Hulíková Tesárková, K. (2016) *Problematika neúspěšných ukončení vysokoškolského studia (drop-outs) v českém kontextu*, [online], Available at: http://kredo.reformy-msmt.cz/download/w-13-3/KREDO_prezentace_150313_2_Hulikova-Tesarkova-Hulik- Hraba.pdf [28 Feb 2018].

Charvát, M., Viktorová, L., Vobořil, L, Tošenová, M. and Opletalová, V. (2014). *Tvorba, administrace a analýza testů studijních předpokladů*. Olomouc: Palacký University in Olomouc.

Jensen, U. (2011) ‚Factors influencing student retention in higher education', *Kamehameha Schools Research & Evaluation*, Vol. 2, pp.1-4.

Kearney, M. S. and Levine, P. B. (2016) ´Income Inequality, Social Mobility, and the Decision To Drop Out of High School´, *NBER Working Paper*, (Spring 2016), pp. 1-73. http://dx.doi.org/10.3386/w20195

Kingston, E. (2008) ‚Emotional competence and drop-out rates in higher education', *Education + Training*, Vol. 50, No. 2, pp. 128-139. https://doi.org/10.1108/00400910810862119

Klaschka J., and Kotrč, E. (2004) *Klasifikační a regresní lesy*. Prague: JČMF.

Komprdová, K. (2012). *Rozhodovací stromy a lesy*. Brno: akademické nakladatelství CERM.

Konečný, T., Basl, J. and Mysliveček, J.. (2010) ‚Přechod mezi střední a vysokou školou a role různých modelů přijímacích řízení', *Czech Sociological Review*, Vol. 46, No. 1, pp. 43–72.

Menclová, M., Pacnerová, H. and Vacek, J. (2008) *Příčiny předčasných odchodů ze středního školního vzdělávání u žáků ze sociokulturně znevýhodněného prostředí, průzkum pro projekt PROPOS*. Prague: Centrum adiktologie a IPPP.

Ministry of Education, Youth and Sports (2014) *Rámec rozvoje vysokého školství do roku 2020*, [online], Available at: http://www.vzdelavani2020.cz/images_obsah/dokumenty/ramec_vs.pdf [26 Feb 2018].

Ministry of Education, Youth and Sports (2017) *Věstník MŠMT*, [online], Available at: http://www.msmt.cz/dokumenty-3/vestnik-msmt-11-12-2017 [20 Feb 2018].

Pikálková, S., Vojtěch, J. and Kleňha, D. (2014) *Úspěšnost absolventů středních škol ve vysokoškolském studiu, předčasné odchody ze vzdělávání*, Prague: Národní ústav pro vzdělávání.

Ptáčková, V., Mazouch, P. and Fischer, J. (2017) *Plýtvá naše společnost lidským kapitálem*?. Prague: University of Economics, Prague.

Rubešová, J. (2009) ‚Souvisí úspěšnost studia na vysoké škole se středoškolským prospěchem?', *Pedagogická orientace*, Vol. 19, No. 3, pp. 89–103.

Řeháková, B. (2000) ‹Nebojte se logistické regrese›, *Sociologický časopis/Czech Sociological Review*, Vol. 36, No. 4, pp. 475-492.

Stoessel, K., Ihme, T. A, Barbarino, M., Fisseler, B. and Stürmer, S. (2015) ‚Sociodemographic Diversity and Distance Education: Who Drops Out from Academic Programs and Why?', *Research in Higher Education*, Vol. 56, No. 3, pp. 228-246. https://doi.org/10.1007/s11162-014-9343-x

Tinto, V. (1975) ‚Dropout from Higher Education: A Theoretical Synthesis of Recent Research', *Review of Educational Research*, Vol. 45, No. 1, pp. 89-125. https://doi.org/10.3102/00346543045001089

Tinto, V. (1997) ‚Classrooms as Communities', *The Journal of Higher Education*, Vol. 68, No. 6, pp. 599-623. https://doi.org/10.2307/2959965

Tinto, V. (1999) ‚Taking Retention Seriously: Rethinking the First Year of College', *NACADA Journal*, Vol. 19, No. 2, pp. 5–9. https://doi.org/10.12930/0271-9517-19.2.5

Tufféry, S. (2011) *Data mining and statistics for decision making*. United Kingdom: Wiley and Sons Ltd.

Vild, J. (2012) *Analýza CHAID jako nástroj vyhodnocování marketingových kampaní*, Prague: University of Economics, Prague.

Vlk, A., Drbohlav, J., Fliegl, T., Hulík, V., Stiburek, Š. and Švec, V. (2017) *Studijní neúspěšnost na vysokých školách: Teoretická východiska, empirické poznatky a doporučení,* Prague: SLON.

Vnoučková, L., Urbancová, H. and Smolová, H. (2017) ‚Factors Describing Students' Perception on Education Quality Standards', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 10, No. 4, pp. 109-115. https://doi.org/10.7160/eriesj.2017.100403

Wolter, S. C., Diem, A. and Messer, D. (2014) ‚Drop-outs from Swiss Universities: An empirical analysis of data on all students between 1975 and 2008', *European Journal of Education*, Vol. 49, No. 4, pp. 471–483. https://doi.org/10.1111/ejed.12096